An introduction to subclasses of basic chordal graphs

Pablo De Caria, Marisa Gutierrez

CONICET/ Departamento de Matemática, Universidad Nacional de La Plata

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Chordal graphs

Every cycle of length at least four has a chord.
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Clique tree

For all $v \in V(G)$, the family $C_v$ of cliques containing $v$ induces a subtree of $T$. 
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Let $S$ be a minimal separator of $G$.

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$S = \{2, 5\}$

$C_S = \{C_2, C_3\}$

$B_S = \{C_2, C_3, C_4\}$
Theorem

A graph $G$ is basic chordal iff for all $S \in S(G)$, $B_S = C_S$. 
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<table>
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<th>$C_S$</th>
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Proposition
Let $G$ be a chordal graph and $V'$ be the set of vertices of $G$ that are not simplicial. Let $G'$ be the graph constructed from $G$ by adding, for each $v \in V'$, a vertex $v^*$ and the edge $vv^*$. Then, $G'$ is basic chordal.
Hereditary basic chordal graphs

A graph $G$ is **hereditary basic chordal (HBC)** if $G$ and all its induced subgraphs are basic chordal.
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Hereditary basic chordal graphs

A characterization of a graph $G$ being a HBC graph:

$G$ is a HBC graph if and only if $G$ is a \{dart, gem\}-free chordal graph.

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An introduction to subclasses of basic chordal graphs
Hereditary basic chordal graphs

1º Characterization

$G$ is a HBC graph $\iff G$ is a \{dart, gem\}-free chordal graph
Other characterizations

The following are equivalent:

- $G$ is a HBC graph
- Every edge of $K(G)$ is in some clique tree of $G$ and no minimal vertex separator of $G$ contains another
- For every triple $C_1, C_2, C_3$ of pairwise intersecting cliques of $G$, $C_1 \cap C_2 = C_1 \cap C_3 = C_2 \cap C_3$
- A clique $C$ intersects a minimal vertex separator $S$ if and only if $S \subseteq C$
- The minimal vertex separators of $G$ are pairwise disjoint
The structure of HBC graphs

**Proposition:** The vertices of a minimal vertex separator $S$ of a $HBC$ graph are twins (have the same closed neighborhood).

**Reason:** Any clique containing a vertex of $S$ contains all the vertices of $S$.

Let $G$ be a $HBC$ graph. The maximal sets of twin vertices of $G$ are minimal vertex separators or consist of simplicial vertices.
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Let $G'$ be the graph whose vertices are the maximal sets of twin vertices, where $A$ and $B$ are adjacent if and only if $G[A \cup B]$ is complete.
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$G'$ is a block graph.

HBC graphs arise from block graphs by replacing its vertices by sets of pairwise adjacent vertices.
Equivalent classes

(4,6)-leaf powers [Brandstädt and Wagner, 2008]

A graph \( G \) is a \((4,6)\)-leaf power if there exists a tree \( T \) whose leaves are the vertices of \( G \) and such that

\[
d_T(v, w) \leq 4 \text{ for all } vw \in E(G).
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d_T(v, w) \geq 6 \text{ for all } vw \not\in E(G).
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2-simplicial powers of block graphs[Brandstädt and Le, 2008]

A graph $G$ is the \textbf{k-simplicial power} of a graph $H$ if $V(G)$ equals the set of simplicial vertices of $H$, and for all distinct vertices $x$ and $y$ of $V(G)$, $xy \in E(G)$ if and only if the distance in $H$ between $x$ and $y$ is at most $k$. 
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Contour vertices and convexity [José Cáceres et al, 2005]

Let \( S \) be a convex set of \( G \) and \( u \in S \). Let \( \text{ecc}_S(u) = \max\{d(u, v) : v \in S\} \).

\( u \) is a contour vertex of \( S \) if \( \text{ecc}_S(u) \geq \text{ecc}_S(v) \) for every neighbor \( v \) of \( u \) in \( S \).
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$G$ is a HBC graph if and only if the set of contour vertices of $S$ equals the set of simplicial vertices of $G[S]$, for every convex set $S$. 
UV, DV and RDV graphs

A clique tree $T$ of $G$ is a....

**UV clique tree:** when $C_v$ induces a path in $T$ for every $v \in V(G)$.

**DV clique tree:** when the edges of $T$ are directed so that $C_v$ induces a directed path in $T$ for every $v \in V(G)$.

**RDV clique tree:** when it is a DV clique tree and it is rooted.
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A graph is UV/DV/RDV if it has a UV/DV/RDV-clique tree.
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Do they have dual classes?

The clique graphs of all $UV$ graphs are also the dually chordal graphs.
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Dually $DV$ graphs: the clique graphs of all $DV$ graphs.

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**Another question**
What is the relationship between the $DV(RDV)$-clique trees of a $DV(RDV)$ graph $G$ and the $DV(RDV)$-compatible trees of $K(G)$?
**Property**

Let $G$ be a $DV(RDV)$ graph. Then, every $DV(RDV)$-clique tree of $G$ is a $DV(RDV)$-compatible tree of $K(G)$. 
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**Sketch of proof**

$DV(RDV)$ graphs are clique-Helly. Hence, every clique of $K(G)$ is of the form $C_v$, for some $v \in V(G)$. Therefore, the cliques of $K(G)$ induce directed paths of every $DV(RDV)$-clique tree of $G$. 

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The converse is not true

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Property

Let $G$ be a basic chordal graph and $T$ be a $DV(RDV)$-compatible tree of $K(G)$. Then, $T$ is a $DV(RDV)$-clique tree of $G$. 

Sketch of proof:

Let $v \in V(G)$. The cliques in $C_v$ are pairwise adjacent in $K(G)$. Let $D$ be a clique of $K(G)$ such that $C_v \subseteq D$. $T[C_v]$ is a subtree of $T$ because $G$ is basic chordal. $T[D]$ is a directed path because $T$ is $DV(RDV)$-compatible. A subtree of a directed path is a directed path.

Conclusion

In basic chordal graphs, the correspondence is not only between its clique trees and the compatible trees of its clique graph but it is stronger in a way that the $DV$ and $RDV$ class trees of both graphs are identical, when they exist.
**Property**

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Let $G$ be a basic chordal graph and $T$ be a $DV(RDV)$-compatible tree of $K(G)$. Then, $T$ is a $DV(RDV)$-clique tree of $G$.

**Sketch of proof:** Let $v \in V(G)$. The cliques in $C_v$ are pairwise adjacent in $K(G)$. Let $D$ be a clique of $K(G)$ such that $C_v \subseteq D$.

$T[C_v]$ is a subtree of $T$ because $G$ is basic chordal. $T[D]$ is a directed path because $T$ is $DV(RDV)$-compatible.

A subtree of a directed path is a directed path.

**Conclusion**

In basic chordal graphs, the correspondence is not only between its clique trees and the compatible trees of its clique graph but it is stronger in a way that the $DV$ and $RDV$ class trees of both graphs are identical, when they exist.
Let us define

**Basic DV graphs**

$DV$ and basic chordal graphs whose $DV$-clique trees are exactly the $DV$-compatible trees of $K(G)$.

**Basic RDV graphs**

$RDV$ and basic chordal graphs whose $RDV$-clique trees are exactly the $RDV$-compatible trees of $K(G)$. 
Let us define

**Basic DV graphs**

$DV$ and basic chordal graphs whose $DV$-clique trees are exactly the $DV$-compatible trees of $K(G)$.

**Basic RDV graphs**

$RDV$ and basic chordal graphs whose $RDV$-clique trees are exactly the $RDV$-compatible trees of $K(G)$.

**Consequences**

- $BASIC \ DV = BASIC \ CHORDAL \cap \ DV$
- $BASIC \ RDV = BASIC \ CHORDAL \cap \ RDV$
Thank you!
An introduction to subclasses of basic chordal graphs