After ten years, we have again the great pleasure to host a new edition of the Latin American Workshop on Cliques in Graphs. The First Latin American Workshop on Cliques in Graphs was held in Río de Janeiro in 2002; the series continued with La Plata/Argentina (2006), Guanajuato/Mexico (2008), Itaipava/Brazil (2010), Buenos Aires/Argentina (2010), and Pirenópolis/Brazil (2012).

Throughout all these years, the number of participants and lectures have increased significantly. In this seventh edition, the Workshop has 6 plenary conferences, 54 contributed talks and the presence of about 90 researchers and students. We are really grateful to all the participants for their contributions, in particular to the invited speakers.

The aim of the Latin American Workshop on Cliques in Graphs is to promote a meeting of researchers in Graph Theory, Algorithms and Combinatorics, specially those working in Graph Operators, Intersection Graphs, Perfect Graphs, and related topics. The main goal in this series of workshops is to strengthen existing collaboration and to promote the creation of new international research groups.

We are really grateful to the members of the Steering and Program Committees and especially to the members of the Organizing Committee, who have worked hard in carrying out the many tasks necessary for successfully holding this meeting.

We thank the financial support given by Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET-Argentina), Universidad Nacional de La Plata (UNLP), Universidad Autónoma Metropolitana of Mexico (UAM) and Consejo Nacional de Ciencia y Tecnología (CONACyT-Mexico) which, through our esteemed colleague Miguel Pizaña, contributed significantly.

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Conference Program

November 8th

- Registration (18:00 - 20:00)
- Welcome Ceremony (20:00)

November 9th

Plenary Talk (8:30 - 9:30)

And/Or-Convexity: a Graph Convexity Based on Processes and Deadlock Models (Fabio Protti).

Room 1 (9:30 - 10:20)

1 Characterization by forbidden induced subgraphs of some subclasses of chordal graphs. S. Nogueira, V. dos Santos.

2 Characterization of the CPT posets in the k-tree class. L. Alcn, N. Gudio, M. Gutierrez.

Room 2 (9:30 - 10:20)

3 The strict terminal connection problem with a bounded number of routers. A. Melo, C. de Figueiredo, U. Souza.


Coffee break (10:20 - 10:40)

Room 1 (10:40 - 11:55)


Room 2 (10:40 - 11:55)

8 On the b-continuity of the lexicographic product of graphs. C. Linhares Sales, L. Sampaio, A. Silva.

9 Parameterized complexity in list coloring problems. R. de Freitas, S. Gama, U. Souza.

10 A linear algorithm for the k-tuple chromatic number of partner limited graphs. F. Bonomo, I. Koch, M. Valencia-Pabon.

Lunch (11:55 - 14:00)

Plenary Talk (14:00 - 15:00)

On Monochromatic Partitions (Maya Stein).

Room 1 (15:00 - 15:50)

11 Integralidad de los modelos arco-circulares unitarios que son minimales. F. Soulignac, P. Terlisky.

12 Characterization and linear-time detection of the minimal obstructions to concave-round graphs and the circular-ones property. M. Safe.

Room 2 (15:00 15:50)


14 Clique Corona Graphs - A Short Survey. V. Levit, E. Mandrescu.

Coffee break (15:50 - 16:10)

Room 1 (16:10 17:50)

15 The Short Block-Move Closest Permutation Problem is NP-Complete. L. Cunha, V. dos Santos, L. Kowada, C. de Figueiredo.

16 On the problem of finding all minimum spanning trees. J. Martinez, R. de Freitas, A. da Silva.

Room 2 (16:10 - 17:50)

18 1-identifying codes on caterpillar graphs. J. Félix, M. Cappelle.

19 Error Correcting Codes and Cliques of Graphs. N. Pedroza, P. Pinto, J. Szwarcfiter.


November 10th

Plenary Talk (8:30 9:30)

Structural results on circle graphs: a survey and the main open problems (Guillermo Durán).

Room 1 (9:30 - 10:20)


23 Sobre el comportamiento del operador diclique. M. Gutierrez, B. Llano, S. Tondato.

Room 2: (9:30 - 10:20)

24 Worst Case Instances for Maximum Clique Algorithms and How to Avoid Them. A. Prusch Zge, R. Carmo.

25 Using PMaxSat Techniques to Solve the Maximum Clique Problem. A. Prusch Zge, R. Carmo, R. Tavares de Oliveira, F. Silva.

Coffee break (10:20 - 10:40)

Room 1 (10:40 - 11:55)

26 Locating-dominating partitions in graphs. I. Pelayo, C. Hernando, M. Mora.


28 On the complexity of $k$-tuple total and total {$k$}-domination in graphs G. Argiroffo, V. Leoni, P. Torres.
Room 2 (10:40 - 11:55)

29 Edge Coloring in Split-Comparability Graphs. J. de Sousa Cruz, C. N. da Silva, S. M. de Almeida.


31 Total Coloring and AVD Total Coloring in Complete Tripartite Graphs. I. R. Tiburcio, S. Morais de Almeida.

Lunch (11:55 - 14:00)

Plenary Talk (14:00 - 15:00)

On the structure of maximal cliques for cocomparability graphs (Michel Habib).

Room 1 (15:00 - 16:15)

32 Caracterización espectral de la composición de ciertos grafos que poseen subgrafos inducidos completos por niveles. L. Medina.

33 Critical Ideals of Digraphs. C. Alfaro, C. Valencia, A. Vázquez-Ávila.

34 $D^L$-integral and $D^Q$-integral graphs. C. M. da Silva, M. Aguiar, A. de Freitas, R. Del-Vecchio.

Room 2 (15:00 - 16:15)


36 On Strong Graph Bundles and Clique Graphs. F. Larrión, M. Pizaña, R. Villarroel-Flores.

37 $K_4$-free clique graphs with unique critical generator and $K_5$-free clique graphs with each triangle contained in at most one $K_4$ with unique critical generator. G. Ravenna, L. Alcón.

Coffee break (16:15 - 16:35)

Visit to the Planetarium (16:40)

Special dinner (21:30)
**November 11th**

**Plenary Talk (8:30 - 9:30)**

On the dichromatic number of planar graphs (Bernardo Llano).

**Room 1 (9:30 - 10:20)**

38 Equitable total coloring of Loupekine Snarks and its products. L. Cordeiro, S. Dantas, D. Sasaki.


**Room 2 (9:30 - 10:20)**

40 On the parameterized complexity of finding all the cliques of a graph. M. Pizaña, I. Robles, L. Taravilse.

41 On the clique behavior’s undecidability for finitely presented graphs. C. Cedillo, M. Pizaña.

**Coffee Break (10:20 - 10:40)**

**Room 1 (10:40 - 11:55)**


43 Characterizing probe unit interval graphs within the class of interval graphs. L. Grippo.

44 A structural characterization of minimal completions from interval graphs to proper interval graphs. N. Pardal, M. Valencia-Pabon, G. Durán.

**Room 2 (10:40 - 11:55)**


46 Bounds on the number of tessellations in graphs. A. Abreu, L. Cunha, L. Kowada, F. Marquezino.

Plenary Talk (14:00 - 15:00)

Cliques and Graph Classes (Martin Milanić).

Room 1 (15:00 - 15:25)

48 The Petersen Graph Synchronizes. E. Canale.

Room 2 (15:00 - 15:25)


Coffee break (15:25 - 15:45)

Room 1 (15:45 - 17:00)


52 Convexidade de Precessão em Digrafos. A. Pereira, C. Centeno.

Room 2 (15:45 - 17:00)


54 Maximal Independent Sets in Simple Graphs. C. Ortiz, M. Villanueva.
Deadlock prevention techniques are essential in the design of robust distributed systems. However, despite the large number of different algorithmic approaches to detect and solve deadlock situations, a quite wide field still remains to be explored in the study of deadlock-related combinatorial properties. In this work, we consider a simplified AND-OR model, where the processes and their communication are given as a graph $G$. Each vertex of $G$ is labelled AND or OR, in such a way that an AND-vertex (resp., OR-vertex) depends on the computation of all (resp., at least one) of its neighbors. We define a graph convexity based on this model, so that a set $S \subseteq V(G)$ is convex if and only if every AND-vertex (resp., OR-vertex) $v \in V(G) \setminus S$ has at least one (resp., all) of its neighbors in $V(G) \setminus S$. We relate some classical convexity parameters to blocking sets that cause deadlock. In particular, we show that those parameters in a graph represent the sizes of minimum or maximum blocking sets, and also the computation time until system stability is reached. Finally, a study on the complexity of combinatorial problems related to such graph convexity is provided. This is joint work with Carlos Vinícius G. C. Lima, Dieter Rautenbach, Uéverton S. Souza, and Jayme L. Szwarcfiter.
On Monochromatic Partitions

Maya Stein *
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Keywords: Complete Graph, Coloring, Monochromatic partition

We survey the field of monochromatic subgraph partitions, whose central problem can be stated as follows: given a complete graph on \( n \) vertices whose edges are colored with \( r \) colors, one would like to find a small set of vertex-disjoint monochromatic subgraphs of a certain type (e.g. cycles) which together cover all the vertices. This problem goes back to work by Gyárfás from the late 1960’s, but its variants have attracted much interest during the past decade. Our survey will cover the most important results and methods of the area.
Structural results on circle graphs: a survey
and the main open problems

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² Universidad de Chile, Chile

Keywords: Intersection graph, Circle graph, Graph subclass

Circle graphs are the intersection graphs of chords on a circle. This class of graphs has been the subject of much study for many years and numerous interesting results have been reported. Many subclasses of circle graphs have been defined and different characterizations were formulated. In this talk, we summarize the most important structural results related to circle graphs and present the main open problems.

This is joint work with Flavia Bonomo, Luciano Grippo, Nina Pardal y Martín Safe.
On the structure of maximal cliques for cocomparability graphs

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IRIF, CNRS & Université Paris Diderot, Paris

Keywords: Cocomparability graph, Interval graph, Clique

A cocomparability graph is a graph whose complement admits a transitive orientation. An interval graph is the intersection graph of a family of intervals on the real line. In this paper, we investigate the relationships between interval and cocomparability graphs. I will first present some recent algorithms we obtained on cocomparability graphs [1, 2]. They show that, for some problems, the algorithm used on interval graphs can also be used with small modifications on cocomparability graphs. Many of these algorithms are based on graph searches that preserve cocomparability orderings.

Then I will propose a characterization of cocomparability graphs via a lattice structure on the set of their maximal cliques. Using this characterization we can prove that every maximal interval subgraph of a cocomparability graph $G$ is also a maximal chordal subgraph of $G$.

This characterization also has interesting algorithmic consequences and we show that a new graph search, namely, Local Maximal Neighborhood Search (LocalMNS) leads to an $O(n + m\log n)$ time algorithm to find a maximal interval subgraph of a cocomparability graph. Similarly, I propose a linear time algorithm to compute all simplicial vertices in a cocomparability graph. In both cases we improve on the current state of knowledge.

References


On the dichromatic number of planar graphs

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Keywords: Planar (di)graph; Dichromatic number; Chromatic number

In this talk, every graph \( G \) is simple, finite and undirected. Let \( D \) be a finite oriented graph (with no symmetric arcs or dicycles, so it can be considered as an orientation of a simple graph). The dichromatic number of \( D \), denoted by \( dc(D) \), is the minimum number of colors in a coloring of the vertices of \( D \) such that each chromatic class induces an acyclic subdigraph of \( D \) (that is, a subdigraph containing no directed cycles). This notion was introduced by V. Neumann-Lara. P. Erdős and V. Neumann-Lara also defined together the dichromatic number of a graph \( G \) as

\[
dc(G) = \max\{dc(\vec{G}) : \vec{G} \text{ is an orientation of } G\}.
\]

The dichromatic number of a digraph has been rediscovered several times since then in terms of quasi colorings, circular chromatic number, or simply, chromatic number of a digraph, among others. One of the most important open conjectures on the dichromatic number is:

**Conjecture 1 (V. Neumann–Lara, 1985)** The vertices of every planar non acyclic oriented graph can be partitioned into two sets such that each set induces an acyclic digraph, that is, \( dc(D) = 2 \) for every planar digraph \( D \). Equivalently, If \( \vec{G} \) is a non acyclic orientation of a planar graph, then \( dc(\vec{G}) = 2 \).

The second part of this conjecture is Conjecture 13.12.6 of the classic book on digraphs by Bang-Jensen and Gutin. In its more general setting, we have

**Conjecture 2** \( dc(G) \leq 2 \) for every planar graph \( G \).

In this form, Conjecture 2 is unsolved problem 26 of the well-known book on graphs by Bondy and Murty. In this talk, we present some results towards the solution of both conjectures.
Cliques and Graph Classes

Martin Milanič
University of Primorska, Koper, Slovenia

Keywords: Clique, Independent set, Graph class

Many graph classes can be defined by imposing conditions that must be satisfied by some or all cliques and/or by some or all independent sets of the graph. Examples of such conditions include: the existence of a partition of the graph’s vertex set into a specified number of cliques and/or independent sets, the existence of a clique or independent set satisfying certain properties, the existence of a linear weight function on the vertices separating the independent sets (or the maximal independent sets) from all other vertex subsets, restrictions on the relations between the sizes of maximal cliques, and restrictions related to the intersections between cliques and independent sets. This talk will survey the basic ideas leading to definitions of such graph classes, relationships between the classes, and connections to other graph theoretic notions. Several structural and algorithmic open questions will also be presented.
Contributed talks

(The order number in the program is the page number)
Characterization by forbidden induced subgraphs of some subclasses of chordal graphs†

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Keywords: Chordal graphs, minimal clique separators, forbidden induced subgraphs

Chordal graphs are the graphs in which every cycle of length at least four has a chord. A graph $G$ is chordal if and only if every minimal vertex separator is a clique. We study subclasses of chordal graphs defined by restrictions imposed on the intersections of its minimal separator cliques. Our goal is to characterize them by forbidden induced subgraphs.

Some of these classes have already been studied, such as chordal graphs in which two minimal separators have intersection if and only if they are equal. Those graphs are known as strictly chordal graphs and were first introduced as block duplicate graphs by Golumbic and Peled [1] and were also considered in [2], based on hypergraph properties. Strictly chordal graphs are exactly the $\{\text{gem, dart}\}$-free graphs [1]. In this work, we study other possible cases of intersection between two minimal separators and the forbidden induced subgraphs of the resulting classes.

References


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Characterization of the CPT posets in the k-tree class

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Keywords: k-tree graphs, posets, containment model

Let $P = (X, P)$ be a partially ordered set or poset. A family $M = (M_x)_{x \in X}$ is a containment model of $P$ if each element $x$ of $X$ can be mapped into a set $M_x$ in such a way that $x < y$ in $P$ $\iff$ $M_x \subset M_y$. If a poset admits a containment model where each set of the family is an interval of the line, then we will say that it is a containment order of intervals, or CI poset for short.

A poset $P$ is a containment order of paths in a tree, or CPT poset for brevity, if it admits a containment model where every $M_x$ is a path of a tree $T$. In [1], we showed the following necessary condition for being a CPT poset:

If $z$ is a vertex of a CPT poset $P$ then $P(D[z])$ is a CI poset. (i)

A graph $G$ is $k$-tree if it can be built recursively, starting with a complete of size $k$ and then, in each step, adding a vertex with exactly $k$ neighbours which induce a complete set. Then we say that a poset $P = (X, P)$ is $k$-tree if its comparability graph $G_P$ is a $k$-tree graph.

In this work we show that $k$-tree graphs admit a recursive construction too. As a consequence of this property, we prove that the necessary condition (i) is also sufficient for the CPT $k$-tree class. In addition, we obtain a characterization by forbidden subposets for this class.

References

The strict terminal connection problem with a bounded number of routers

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\textit{Keywords: Terminal connection tree; Steiner tree; Disjoint paths}

A connection tree $T$ of $G$ on a terminal set $W \subseteq V(G)$ is a tree subgraph of $G$ such that $W \subseteq V(T)$ and every leaf of $T$ is contained in $W$. The vertices of $V(T) \setminus W$ with degree exactly 2 in $T$ are called linkers and the vertices of $V(T) \setminus W$ with degree at least 3 in $T$ are called routers. Motivated by applications in networks, Dourado et al. [1] proposed the \textsc{Terminal connection problem} (TCP): given a connected graph $G$, a set $W \subseteq V(G)$, and two non-negative integers $\ell$ and $r$; we ask if $G$ admits a connection tree on $W$ with at most $\ell$ linkers and $r$ routers. The TCP was proved to be NP-complete even when either $\ell$ or $r$ is bounded by a constant [1]. The \textsc{Strict TCP} (S-TCP) further requires that every vertex of $W$ is a leaf of $T$, and it was proved to be NP-complete even when $\ell$ is bounded by a constant [2].

We analyse the S-TCP when $r$ is bounded by a constant. We prove that the S-TCP is in P for $r = 0$ by a Karp reduction to \textsc{Shortest path}, for $r = 1$ by a Turing reduction to \textsc{Min-sum st-disjoint paths}, and for $r = 2$ when we require that the two routers are adjacent. Conversely, we show the NP-completeness of the case $r = 2$ when $W = W_1 \cup W_2$ and all vertices of $W_i$ are routed by the same router, by a Karp reduction from $k + 1$ \textsc{disjoint paths between two pairs of vertices}.

\textbf{References}


A watermarking scheme for structured programs

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Keywords: structured programming, software protection, control flow graph

Watermarking a digital object is the act of embedding into the object some (often surreptitious) data meant to disclose its authorship/ownership. Graph-based watermarking schemes consist of encoding/decoding algorithms (codecs) that translate the identification data onto (and back from) some special kind of graph, and embedding/extracting algorithms to insert/recognize the watermark into/from the program (CFG).

A good watermarking scheme must have the following properties: the watermarking should not adversely affect the size and execution time of the marked program (small size and efficiency); the watermark must be recognized even after the watermarked program has been subjected to an attack (resilience); the codec should generate reasonably different graphs to encode the same key (diversity); and the watermark should be well disguised into the code (stealthiness), preventing that it is found and removed after some reverse engineering by a malicious party.

We propose a new codec for graph-based software watermarking which presents the desired properties: resiliency, small size, efficiency, diversity, and stealthiness. Moreover, it has an important characteristic, which to our knowledge cannot be found in previous watermarking schemes: our graphs are inserted into the CFG by adding structured dummy code, i.e., the watermarked CFG of a structured program is also structured [1].

References

On the $P_3$-Hull Number of the Cartesian Product of Graphs $^1$

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Keywords: $P_3$-convexity, $P_3$-hull number, Cartesian product.

Let $G$ be a finite, simple, and undirected graph and let $S$ be a set of vertices of $G$. If every vertex having two neighbors inside $S$ is also in $S$, then $S$ is $P_3$-convex. The $P_3$-convex hull $H(S)$ of $S$ is the smallest $P_3$-convex set containing $S$. If $H(S) = V(G)$ we say that $S$ is a $P_3$-hull set of $G$. The cardinality $h(G)$ of a minimum $P_3$-hull set in $G$ is called the $P_3$-hull number of $G$.

Let $G$ be a graph and let $K_n$ be a complete graph on $n$ vertices. In $[1]$ has been proved that the $P_3$-hull number can be determined in polynomial time for complementary prisms. In this paper we determine the $P_3$-hull number of the Cartesian product $G \square K_n$ and we present upper and lower bounds for the $P_3$-hull number of the Cartesian product $G \square H$ of general graphs $G$ and $H$.

References


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On Convex Partitions of Graphs in the Triangle-Path Convexity

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Keywords: Triangle-path convexity, Partition into convex sets

A triangle path is a path \((v_1, \ldots, v_k)\) such that there is no edge between \(v_i\) and \(v_j\) if \(|i - j| > 2\). In the triangle-path convexity, the convex sets are those where every triangle path between vertices of \(S\) contains only vertices of \(S\). The complexity of many fundamental convexity parameters on the triangle-path convexity has been studied previously in [3].

We consider the PARTITION INTO CONVEX SETS problem in which, given a graph \(G\) and an integer \(p\), the question is to decide whether \(V(G)\) can be partitioned into \(p\) convex sets. We show that PARTITION INTO CONVEX SETS can be solved in polynomial time for fixed \(p\) in the triangle-path convexity for graphs without clique separators, contrasting with results for other convexities, in which the problem is NP-complete even for fixed \(p\) [1, 2].

References


1Partially supported by CNPq and FAPEMIG
Convex \( p \)-partitions and convex \( p \)-covers

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Martín D. Safe \(^1,2\) Vinícius F. dos Santos \(^3\)
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Keywords: convex covers, convex partitions, graph convexity

Convexities in graphs have been widely studied in the last years. Several articles can be found in the literature dealing with algorithmic and complexity issues of parameters related to convexities in graphs. Interesting enough is to compare the behavior of these parameters under different convexities from an algorithmic and complexity point of view.

If \( \mathcal{P} \) is a set of paths in a graph \( G \) and \( \mathcal{C} \) is the collection of all subsets \( S \) of \( V(G) \) such that, for every \( P \in \mathcal{P} \) whose end-vertices belong to \( S \), every vertex of \( P \) belongs to \( S \), then \( \mathcal{C} \) is a graph convexity. The monophonic convexity, the \( P_{3}^{*} \)-convexity, and the \( P_{3} \)-convexity are the convexities whose convex sets are generated by induced paths, induced paths of length two, and paths of length two of the graph, respectively. The convex sets of a graph \( G \) under the digital convexity are those sets \( S \) of \( G \) such that for every vertex \( v \in V(G) \), if \( N_{G}[v] \subseteq N_{G}[S] \), then \( v \in S \).

Given a graph \( G \), the problems of partitioning \( V(G) \) into \( p \) convex sets and of covering \( V(G) \) by \( p \) convex sets were introduced in [1] and [2], respectively. In this work we present some algorithmic and complexity results for both problems under all of these convexities.

References


On the $b$-continuity of the lexicographic product of graphs

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Keywords: $b$-chromatic number, $b$-continuity, $b$-homomorphism, chordal graphs, $P_4$-sparse graphs

A $b$-coloring of the vertices of a graph is a proper coloring where each color class contains a vertex which is adjacent to each other color class. The $b$-chromatic number of $G$ is the maximum integer $\chi_b(G)$ for which $G$ has a $b$-coloring with $\chi_b(G)$ colors. A graph $G$ is $b$-continuous if $G$ has a $b$-coloring with $k$ colors, for every integer $k$ in the interval $[\chi(G), \chi_b(G)]$. It is known that not all graphs are $b$-continuous. Here, we investigate whether the lexicographic product $G[H]$ of $b$-continuous graphs $G$ and $H$ is also $b$-continuous. Using homomorphisms, we provide a new lower bound for $\chi_b(G[H])$, namely $\chi_b(G[K_t])$, where $t = \chi_b(H)$, and prove that if $G[K_\ell]$ is $b$-continuous for every positive integer $\ell$, then $G[H]$ admits a $b$-coloring with $k$ colors, for every $k$ in the interval $[\chi(G[H]), \chi_b(G[K_\ell])]$. We also prove that $G[K_\ell]$ is $b$-continuous for every positive integer $\ell$ whenever $G$ is a $P_4$-sparse graph, and we give further results on the $b$-spectrum of $G[K_\ell]$ when $G$ is chordal. Finally, we determine the value of $\chi_b(T[K_\ell])$ when $T$ is a tree.
Graph coloring problems with special constraints, add a set of additional conditions on how the colors must be assigned to the vertices, edges or both. Among these problems, the list coloring problem consists in finding a proper coloring of a graph $G = (V, E)$ in which, to each vertex $v \in V(G)$, a list of allowed colors $L(v)$ is associated. The list coloring problem was studied by Vizing and by Erdos, Rubin and Taylor, simultaneously.

Furthermore, recently parameterized complexity has presented several results involving coloring problems such as list coloring, pre-coloring extension and choosability in graphs. A parameterized problem $\Pi$ is informally defined by: an instance, a parameter and a question. A parameterized problem $\Pi(S)$ belongs to the FPT class if there is an algorithm to solve $\Pi(S)$ in time $f(S).n^c$, where $n$ is the size of the input, $c$ is a constant and $f$ a function. The $W$ hierarchy is a collection of computational complexity classes. A parameterized problem is in the class $W[i]$ if every parameterized instance can be transformed in FPT-time to a combinatorial circuit that has weft at most $i$. We know that list coloring is $W[1]$-Hard even when parameterized by the vertex cover. An overview of the literature will be presented.

In this work, we analyze the case where there are restrictions in the size (lower and upper bounds) and type of elements (sequential values) in color lists, as the $(\gamma, \mu) - coloring$ problem. In the constraint satisfaction community in Artificial Intelligence, the list coloring with intervals of colors would be called convex constraints. Finally, we can already say that list coloring is FPT when parameterized by the cover vertex and the maximum size of the lists. The motivation of this result is that the problem is $W[1]$-hard when parameterized by the vertex cover and NP-complete even when the list size is bounded by a constant.
A linear algorithm for the $k$-tuple chromatic number of partner limited graphs

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Keywords: partner limited graphs, $k$-tuple coloring, $k$-chromatic number, lexicographic product of graphs.

In the $k$-tuple coloring problem, we aim to assign sets of colors of size $k$ to the vertices of a graph $G$, so that the sets which belong to adjacent vertices of $G$ have empty intersection, and the total number of colors used is minimum. This minimum number of colors is called the $k$-tuple chromatic number. The problem, introduced independently by Stahl and Bollobás and Thomason in the late seventies, has applications in mobile radio frequency assignments, fleet maintenance, task assignments, and traffic phasing. We present in this work a linear algorithm for computing the $k$-tuple chromatic number of partner limited graphs, a graph family that generalizes many well-known classes of graphs with “few” induced $P_4$s, including cographs, $P_4$-sparse and $P_4$-tidy graphs.
Integralidad de los modelos arco-circulares unitarios que son minimales

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Keywords: modelos arco-circulares, modelos minimales, modelos enteros

Un modelo arco-circular (CA) es un par $\mathcal{M} = (C, \mathcal{A})$ tal que $C$ es un círculo y $\mathcal{A}$ es una familia finita de arcos de $C$. Suponemos que $C$ tiene un punto especial, denotado por 0. Cuando 0 no pertenece a ningún arco de $\mathcal{A}$, decimos que $\mathcal{M}$ es de intervalos (IG). Los modelos UCA y UIG son aquellos modelos CA e IG cuyos arcos tienen todas la misma longitud, respectivamente. Dos modelos CA son equivalentes cuando los extremos de sus arcos aparecen en el mismo orden en un recorrido de sus círculos desde 0.

Dados $\ell > 0$ y $d > 0$, un modelo IG $\mathcal{M}$ es $(\ell, d)$-IG cuando los arcos tienen longitud $\ell$ y cada par de extremos está a distancia al menos $d$. Decimos que $\mathcal{M}$ es $(\infty, d)$-minimal cuando, para todo modelo $(\ell', d)$-IG $\mathcal{M}'$ equivalente, ocurre que: 1. $\ell \leq \ell'$ y 2. $s_i \leq s_i'$ para $1 \leq i \leq n$, donde $s_i$ (resp. $s_i'$) es la posición del $i$-ésimo extremo de $\mathcal{M}$ (resp. $\mathcal{M}'$) desde 0. Esta definición fue propuesta en 1990 por Pirlot, quien demostró que (a) todo modelo UIG $\mathcal{M}$ es equivalente a un modelo $(\infty, d)$-minimal y (b) $\ell$ es múltiplo de $d$ cuando $\mathcal{M}$ es $(\infty, d)$-minimal.

Dados $c$, $\ell$ y $d$ positivos, un modelo CA $\mathcal{M}$ es $(c, \ell, d)$-CA cuando el círculo tiene circunferencia $c$, los arcos tienen longitud $\ell$ y los extremos están a distancia al menos $d$. Considerando que el punto 0 no juega ningún rol en los modelos CA, decimos que $\mathcal{M}$ es $d$-minimal cuando, para todo modelo $(c', \ell', d)$-CA equivalente, 1. $\ell \leq \ell'$ y 2. $c \leq c'$. Vale notar que todo modelo PIG $(\infty, d)$-minimal es $d$-minimal; la recíproca es falsa. El término $d$-minimal fue propuesto por Soulignac para demostrar que todo modelo UCA es similar a un modelo $d$-minimal y conjecturar que $\ell$ y $c$ son múltiplos de $d$.

En este trabajo demostramos esta conjectura, a la vez que obtenemos algoritmos más eficientes para encontrar modelos $d$-minimales.
Characterization and linear-time detection of the minimal obstructions to concave-round graphs and the circular-ones property

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Keywords: circular-ones property, concave-round, minimal obstructions

A graph is concave-round if its vertices can be circularly enumerated so that the closed neighbourhood of each vertex is an interval in the enumeration. In this work, we obtain the minimal forbidden induced subgraphs for the class of concave-round graphs, solving a problem posed in [1]. We also show that it is possible to find, in linear time, one such forbidden induced subgraph in any non-concave-round graph. As part of the analysis, building upon results in [3, 4] we obtain the minimal forbidden submatrices for the circular-ones property for rows (reported as unknown in [2]) and for rows and columns and, also for each of the two variants of the property, a linear-time algorithm which finds a corresponding circular-ones arrangement or a minimal forbidden submatrix.

References


About some special classes of well-covered graphs

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Keywords: Well-covered graphs. Split graphs. Characterization.

A graph is well-covered if all of its maximal independent sets have the same cardinality. In other words, every maximal independent set is maximum. Well covered graphs (also known in literature as unmixed) were introduced in 1970 by [2]. An important algorithmic property of well-covered graphs is that the polynomial greedy algorithm for producing a maximal independent set always produces a maximum independent set when applied to well-covered graphs.

A split graph [1] is a graph such that its vertex set can be partitioned into one stable set and one clique.

In this paper we characterize well-covered split graphs (graphs which are split and well-covered). We also show that the degree sequence of these graphs is well defined. In addition we give partial conditions for a graph such that its vertex set can be partitioned into one independent set and two cliques to be well-covered.

References


Clique Corona Graphs - A Short Survey

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Keywords: corona, well-covered graph, independence polynomial

For a graph $G$, the independence number $\alpha(G)$ is the size of a maximum independent set. $G$ is well-covered if all its maximal independent sets are of the same size (M. D. Plummer, 1970). If, in addition, $G$ has no isolated vertices and its order is equal to $2\alpha(G)$, then $G$ is a very well-covered graph (O. Favaron, 1982). A well-covered graph (of order $\geq 2$) is $1$-well-covered if the deletion of every vertex of the graph leaves a graph which is well-covered as well (J. W. Staples, 1975).

Let $s_k$ be the number of independent sets of size $k$ in $G$. The generating function $I(G; x) = s_0 + s_1 x + s_2 x^2 + \cdots + s_{\alpha(G)} x^{\alpha(G)}$ is the independence polynomial of $G$ (I. Gutman and F. Harary, 1983). If there is an index $k$ such that $s_0 \leq \cdots \leq s_{k-1} \leq s_k \geq s_{k+1} \geq \cdots \geq s_{\alpha(G)}$, then $I(G; x)$ is unimodal.

Let $\mathcal{H} = \{H_v : v \in V\}$ be a family of graphs indexed by the vertex set $V$ of the graph $G$. The corona graph $G \circ \mathcal{H}$ is the disjoint union of $G$ and all $H_v, v \in V$, with additional edges joining each vertex $v$ of $G$ to all the vertices of the corresponding $H_v$ (R. Frucht and F. Harary, 1970). If every $H_v$ is a complete graph, then $G \circ \mathcal{H}$ is a clique corona graph.

Corona graphs have attracted reasonable attention in the literature dealing with vertex coloring, star coloring, bandwidth, thoughness, Szeged index, Zagreb indices, Merrifield-Simmons index, adjacency spectrum, Laplacian spectrum, etc.

In this survey we present a number of findings, conjectures, and open problems, where clique corona graphs are essentially involved. For instance, it happens, when:

- $G$ is well-covered, very well-covered or $1$-well-covered;
- $I(G; x)$ is partially unimodal or it has only real roots.
The Short Block-Move CLOSEST PERMUTATION PROBLEM is NP-Complete

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Keywords: Closest Permutation, NP-Completeness, Hamming distance, Short block-move distance

The CLOSEST OBJECT PROBLEM aims to find one object in the center of all others. It was studied for strings with respect to the Hamming distance, where the HAMMING CLOSEST STRING PROBLEM was determined to be NP-Complete. The CLOSEST PERMUTATION PROBLEM (CPP) was also studied, since permutations are the natural restrictions to general strings, and we have proved that the BLOCK INTERCHANGE–CPP is NP-Complete [1].

We consider a restricted form of the block-interchange, called short block-move, proposed by Heath and Vergara [2], defined by exchanging two contiguous blocks of elements of total length at most 3, for which the computational complexity of the distance problem is open. We provide sufficient conditions to determine the short block-move distance by showing that the optimal sorting sequence of short block-moves of a given permutation can be obtained by sorting each connected component separately on the permutation graph, and we prove that SHORT BLOCK MOVE–CPP is NP-Complete.

References

On the problem of finding all minimum spanning trees

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Keywords: algorithms, computational complexity, graph theory, minimum spanning trees.

Let $G$ be an undirected weighted graph with $n$ vertices and $m$ edges. We say that $T$ is a spanning tree of $G$ if it is a subgraph that connects all vertices of $G$ and does not contain any cycle. $T$ is a minimum spanning tree (MST) if the sum of the weights of the edges is the smallest among all spanning trees. There are classic polynomial algorithms that find a MST in $O(n \log n)$, as Prim and Kruskal algorithms. However, such algorithms retrieve only one MST among the many possible, and therefore comes up the need to explore algorithms that can enumerate all possible MST’s for a graph. One possible solution would be to enumerate all spanning trees and provide only those of minimum cost, but that would demand a high unnecessary computational cost.

Our work consists of a theoretical analysis about algorithmic strategies and properties applied in general graphs to solve the problem of finding all minimum spanning trees and also to define the computational complexity of the algorithms studied. We also checked the behavior of such algorithms in specific graph classes.

![Figure 1](image-url)  
(1) Graph $G$. (2) All spanning trees of $G$. The MSTs are inside the rectangular box.
Sandwich Problems and Structural Properties of the Two Forbidden Four-vertex Subgraphs Classes

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Keywords: graph sandwich problem; forbidden induced subgraphs.

The Π Graph Sandwich Problem (GSP) introduced by Golumbic and Shamir in 1993 consists in determining, for a pair of graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$ with $E^1 \subseteq E^2$, whether there exists a graph $G = (V, E)$ that satisfies property Π and $E^1 \subseteq E \subseteq E^2$. The GSP has attracted much attention because of many applications, so several sandwich problems were considered for different graph classes, for instance: interval graph, unit interval graph, permutation graph and comparability graph sandwich problems are all $NP$-complete, while the split graph, threshold graph and cograph sandwich problems are in $P$.

Dantas et al. (2011, 2015) completely classified the complexity of the $F$-free GSP when $F$ is a four-vertex subgraph. Motivated by a question proposed by M. C. Golumbic about the complexity status of the GSP of the well known trivally perfect graph class, we study several classes with two forbidden four-vertex induced subgraphs and determine that: the problem is in $P$ for {$C_4, P_4$}-free (trivially perfect), {claw, $P_4$}-free, {paw, $P_4$}-free, {diamond, $P_4$}-free, {diamond, $K_4$}-free, {diamond, $C_4$}-free, {diamond, paw}-free, {C4, paw}-free, {claw, paw}-free, {claw, paw}-free, {paw, paw}-free, {4$K_1, K_4$}-free, {claw, claw}-free and {C4, 2$K_2$}-free graphs, while it is $NP$-complete for {$K_4, C_4$}-free, {$K_4, paw$}-free, {4$K_1, paw$}-free and {2$K_2, paw$}-free graphs. In addition, we show structural properties for several classes with two forbidden four-vertex induced subgraphs.

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1-identifying codes on caterpillar graphs

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\textit{Keywords: Graph Theory, Identifying Codes, Caterpillar}

Let $G = (V, E)$ be a connected undirected graph, $C \subseteq V$ be a subset of $V$, and $r \geq 1$. For any $v \in V$, let $B_r(v)$ be the ball of radius $r$ centered at $v$, i.e., the set of all vertices within distance $r$ from $v$. If for all vertices $v \in V$, the sets $B_r(v) \cap C$ are all nonempty and distinct, then we say that $C$ is a $r$-identifying code.

Identifying codes were introduced by Karpovsky, Chakrabarty and Levitin in 1998. The motivation comes from fault diagnosis in multiprocessor systems. Finding identifying codes of minimum cardinality has been proved to be NP-hard by Charon in 2003. Hence, many authors have directed their study of identifying codes at many different classes of graphs. In this paper, we investigate 1-identifying codes on caterpillars, which are trees where all vertices are within distance 1 from a central path $P$.

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Error Correcting Codes and Cliques of Graphs

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Keywords: error correction, codes, clique, graphs

Error correcting codes are widely used in communications and electronic equipments. Traditionally, codes with fixed length have been used. From the last few years, error correcting codes with variable length (VLEC) started to be considered, such as in [1, 2].

The construction of optimum error correcting codes, both with fixed and variable word length, using a minimum amount of bits is still an unsolved problem.

In our attempt to solve this issue, we developed an optimum way to concatenate two fixed length binary codes to build a VLEC code. Based on this technique, we were able to discover a family of VLEC codes which has a lower cost than the corresponding fixed length codes and those proposed in the literature.

Finally, we describe the problem of determining the maximum number of words in a code with distance \(d\) and word length \(n\) as the problem of finding the maximum clique of the graph \(G = K_{2^n} - E[Q_n^{d-1}]\), where \(K_{2^n}\) is the complete graph with \(2^n\) vertices and \(Q_n^{d-1}\) the \((d-1)\)-th power of the n-cube.

References


Algorithms for Deadlock Resolution in Subcubic Graphs

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Keywords: Deadlock; Computational complexity; Subcubic graph.

A deadlock occurs in a distributed computation when a set of processes wait indefinitely for resources from each other. Systems that perform a distributed computation are usually represented by wait-for graphs, where the behavior of each process is determined by a deadlock model, which defines the rules of a distributed computation by determining how processes can become executable. Carneiro et al. [1] proposed Deadlock-Resolution(OR, Vertex), a NP-Hard problem where the first parameter, ‘OR’, indicates the deadlock model of the wait-for graph. The second, ‘Vertex’, indicates that a minimum number of vertices has to be removed in order to solve all the deadlocks in the input graph.

This work proposes two algorithms for subcubic graphs by analyzing all vertices based on their in- and out-degree. First, we prove that a knot (structure that characterizes deadlocks) can be eliminated with only one vertex removal. When solving a knot, at most one new knot may appear in the graph, which can be eliminated by removing one other vertex that does not create new knots. Thus, we get a fast 2-approximative algorithm.

After that, we show that, using (f, g)-semi-matching [2], we can find in polynomial time the minimum number of vertices that must be removed from a subcubic graph to make it become knot-free.

References

Optimal Edge Fault-Tolerant Embedding of a Star over a Cycle

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Keywords: Graph Embedding, Multilayer Networks, Routing.

An embedding of a guest graph $G$ over a host graph $H$, such that the vertices of $G$ are included in the vertices of $H$, consists of a mapping $\rho$ that associates every edge $e = xy$ in $G$ to an $x-y$-path $\rho(e)$ in $H$. Given an edge $f$ in $H$, the number of edges $e$ in $G$ such that $f$ belongs to $\rho(e)$ is called the (edge) congestion of $f$. The length of $\rho(e)$ is called the dilatation of $e$. The sum of all the dilatations is the cost of the embedding. The removal of an edge $f$ of $H$ gives rise to a surviving graph $G_f$, consisting of the guest graph without those edges that cross $f$, i.e., $G_f = G - \{e : f \in \rho(e)\}$.

Given positive integers $n$, $b$, and an $n$ nodes graph $H$ with a distinguished vertex $v$, we are facing the problem of finding a graph $G \subset K_n$ with minimum cost embedding over $H$, in such a way that for each surviving graph $G_f$, there exists another upper embedding of the complete graph $K_{1,n}$ over $G_f$ that keeps the congestion of every $e$ in $G_f$ below $b$, while pairs the vertex of degree $n$ with $v$.

The problem has direct applications to the design of resilient Internet-Service-Providers backbones; specifically to connect customer aggregation points towards data-centers or other prominent nodes. This work presents a general lower bound for the optimal cost of such a problem, and also introduces a family of graphs whose minimal embeddings match that bound for all values of $n$ and $b$, when the host graph is the cycle $C_n$, thus determining a family of optimal solutions for these instances.
Biclique Graphs of a Subclass of Split Graphs

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Keywords: Bicliques; Biclique graphs; split graphs.

A biclique of a graph is a vertex set that induces a maximal complete bipartite subgraph. The biclique graph of a graph $G$, denoted by $KB(G)$, is the intersection graph of the bicliques of $G$. Let $H = (K \cup S, E)$ be a split graph such that $K$ is the complete part and $S$ is the set of satellites. Each vertex of $K$ is adjacent to some satellites of $S$. Let $S(v)$ be the set of satellites of vertex $v \in K$.

We study the recognition problem of biclique graphs of some classes of graphs $\mathcal{A}$, that is, $KB(\mathcal{A})$. Particularly, in this work we study the biclique graphs of a subclass of split graphs called nested separable split graphs (NSS). These graphs are split graphs such that, for every vertex $u \in K$, there is a vertex $v \in K$ such that $S(u) \cap S(v) = \emptyset$ and for every pair $u, v \in K$, if the set of satellites of one is not included in the other, then $S(u) \cap S(v) = \emptyset$. The class of NSS includes Threshold graphs.

We prove that the problem is polynomial by presenting a polynomial time algorithm to decide if a given graph $G$ is the biclique graph of some NSS graph. The algorithm builds a candidate $H$ such that if $G$ is the biclique graph of some NSS graph, then $G$ is isomorphic to $KB(H)$. The idea to construct $H$ is the following: based on the degrees of the vertices of $G$, it finds the vertices of $G$ that are the only candidates to represent a particular family of bicliques of $H$, if such $H$ exists. With that information, it constructs the different sets of vertices of the complete part of $H$ with the same set of satellites. Then, using some properties of the found bicliques, it gives the only possible inclusion order for those sets. Finally, the candidate $H$ is built.

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Sobre el comportamiento del operador diclique

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**Keywords:** diclique, convergencia, divergencia y periodicidad

Sean $D = (V, A)$ un digrafo y $X$ e $Y$ subconjuntos no vacíos de $V$ no necesariamente disjuntos.

Un *disimplex* $K(X, Y)$ de $D$ es un subdigrafo cuyo conjunto de vértices es $X \cup Y$ y su conjunto de arcos es $\{(x, y) : x \in X \land y \in Y\}$ (no se consideran los lazos si $X \cap Y \neq \emptyset$).

Un disimplex es un *diclique* si no es subdigrafo propio de ningún disimplex.

Prisner introdujo el concepto de *grafo diclique (operador diclique)* $\vec{k}(D)$, definido de modo que $V(\vec{k}(D)) = \{K(X, Y) : K(X, Y) \text{ es un diclique de } D\}$ y $A(\vec{k}(D)) = \{(K(X, Y), K(X', Y')) : Y \cap X' \neq \emptyset\}$.

Como ocurre con el operador clique en grafos, resulta de interés conocer el comportamiento al iterar el operador diclique (divergencia, convergencia y periodicidad).

Un digrafo $D$ se dice *$\vec{k}$-divergente* si $|V(\vec{k}^n(D))|$ no es acotado. En caso contrario se dice que es *$\vec{k}$-convergente*.

Un digrafo $D$ se dice *$\vec{k}$-periódico* si existe $n \in N$ tal que $\vec{k}^n(D) \approx D$.

En este trabajo presentamos una familia de digrafo $\vec{k}$-convergentes, un par de grafos $\vec{k}$-periódicos distintos a los reportados en la literatura y una familia de digrafo $\vec{k}$-divergentes.

Los resultados teóricos de este trabajo han sido inspirados en la teoría para grafos divergentes que comenzó V. Neumann-Lara en los 70’s y desarrollada posteriormente por él, P. Larrión y M. A. Pizaña.
Worst Case Instances for Maximum Clique Algorithms and How to Avoid Them

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Keywords: maximum clique, branch and bound, worst-case analysis

The Maximum Clique problem (MC) is the problem of finding a clique of maximum size on a given graph. The problem is $\mathcal{NP}$–hard and approximation to within a factor of $O(n^{1-\varepsilon})$ (where $n$ is the number of vertices) remains $\mathcal{NP}$–hard for all $\varepsilon > 0$.

In spite of these discouraging theoretical facts, several algorithms for the exact solution of MC are reported to solve instances of practical interest and considerable size quite satisfactorily. The best performing (from an experimental point of view) among these algorithms are based on the branch and bound technique and compute coloring(s) of the graph to be used as a bound on the size of the maximum clique.

From an experimental point of view, these algorithms display a behavior that seems sub-exponential. The work in [1] introduces an infinite class of graphs for which such branch and bound algorithms using coloring for bounding must take an exponential ($\Omega(2^{n/5})$) number of steps.

We show that the class of graphs introduced in [1] can be easily extended to a broader class for which algorithms for MC such as those discussed above must take exponential time. We also show that it is easy to detect these graphs in polynomial time and show how to adapt existing branch and bound algorithms for MC so that this class of graphs can be dealt with in polynomial time.

References

Using PMaxSat Techniques to Solve the Maximum Clique Problem

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Keywords: branch and bound, maxsat solvers, experimental analysis

The Maximum Clique problem (MC) is the problem of finding a clique of maximum size on a given graph. The Partial Maximum Boolean Satisfiability problem (PMaxSat) is the problem of finding an assignment that maximizes the number of satisfied clauses in a given Boolean formula in conjunctive normal form, with the constraint that mandatory clauses must be satisfied. Both MC and PMaxSat are \( \mathcal{NP} \)-hard problems, and there are branch and bound algorithms in the literature for the exact solution of either one.

Due to some simple reductions between the problems, a MC instance can be easily encoded into a PMaxSat formula and, so, a MC instance can be solved using any PMaxSat solver. It has been claimed in the literature that solvers can not match the performance of a dedicated algorithm, but there is a lack of data to support this statement for current solvers and algorithms.

Considering these encodings, bounding techniques from algorithms for PMaxSat can be adapted and used directly in branch and bound algorithms for MC. In the recent years, several algorithms for MC were proposed using techniques taken from PMaxSat solvers, and these algorithms are experimentally shown to be competitive in widely used benchmarks.

In this work, we compare the solution of MC using dedicated branch and bound algorithms and PMaxSat solvers. Instances including random and DIMACS graphs were solved with both approaches, and experimental results are shown and discussed. We also analyze how decisions made by PMaxSat solvers relate to branching in dedicated algorithms. The main objective of this work is to gain insights in how decisions made by PMaxSat solvers can be interpreted directly in the graph, and what advances in dedicated algorithms for MC can still be expected from adapting PMaxSat techniques.
Locating-dominating partitions in graphs

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Keywords: dominating partition; locating partition; coloring-locating partition; coloring-locating-dominating partition.

Let $G = (V, E)$ be a connected graph of order $n$. Let $\Pi = \{S_1, \ldots, S_k\}$ be a partition of $V$. Let $r(v|\Pi)$ denote the vector of distances between a vertex $v \in V$ and the elements of $\Pi$, that is, $r(v, \Pi) = (d(v, S_1), \ldots, d(v, S_k))$. The partition $\Pi$ is called a locating partition of $G$ if, for every pair of distinct vertices $u, v \in V$, $r(u, \Pi) \neq r(v, \Pi)$. A locating partition $\Pi$ is called a metric-locating-dominating partition (an MLD-partition for short) of $G$ if it is also dominating, i.e., if, for every $i \in \{1, \ldots, k\}$ and for every vertex $v \in S_i$, $d(v, S_j) = 1$, for some $j \in \{1, \ldots, k\}$. An MLD-partition $\Pi$ is called a locating-dominating partition (an LD-partition for short) of $G$ if, for every $i \in \{1, \ldots, k\}$ and for every pair of distinct vertices $v, w \in S_i$, $d(v, S_j) = 1$ and $d(w, S_j) > 1$, for some $j \in \{1, \ldots, k\}$.

An MLD-partition $\Pi$ is called a coloring-MLD-partition of $G$ if all the parts of $\Pi$ are independent sets. An LD-partition $\Pi$ is called a coloring-LD-partition of $G$ if all the parts of $\Pi$ are independent sets.

The minimum cardinality of an MLD-partition (resp. coloring-MLD-partition) of $G$ is denoted by $\eta_p(G)$ (resp. $\eta^i_p(G)$). The minimum cardinality of an LD-partition (resp. coloring-LD-partition) of $G$ is denoted by $\lambda_p(G)$ (resp. $\lambda^i_p(G)$).

We undertake the study of these four parameters, both separately, and also all together. First, we evaluate the sequence $(\eta_p, \lambda_p, \eta^i_p, \lambda^i_p)$ for the main basic graph families, such as cliques, cocliques, paths, cycles, stars and other families of trees. Then, we obtain some interrelations and realization theorems involving these parameters, mainly when considering trees and other bipartite families. Finally, we approach the equations $\rho(G) = n$, $\rho(G) = n-1$ and $\rho(G) = n-2$, where $\rho \in \{\eta_p, \lambda_p, \eta^i_p, \lambda^i_p\}$.
A sequence of vertices in a graph $G$ without isolated vertices is called a total dominating sequence if every vertex in the sequence totally dominates at least one vertex that was not totally dominated by preceding vertices in the sequence and, at the end all vertices of $G$ are totally dominated (by definition a vertex totally dominates its neighbors). The maximum length of a total dominating sequence is called the Grundy total domination number of $G$, or $\gamma_{tgr}(G)$, as introduced in [B. Brešar, M. A. Henning, and D. F. Rall, Total dominating sequences in graphs, Discrete Math. 339 (2016), 1165–1676]. In this paper we continue the investigation of this concept, mainly from the algorithmic point of view. While it was known that the decision version of the problem is NP-complete in bipartite graphs, we show that this is also true when restricted to the class of split graphs. A linear time algorithm for determining the Grundy total domination number of an arbitrary tree $T$ is presented, based on the formula $\gamma_{tgr}(T) = 2\tau(T)$, where $\tau(T)$ is the vertex cover number of $T$. A similar efficient algorithm is presented for bipartite distance-hereditary graphs. Using the modular decomposition of a graph, we present a frame for obtaining polynomial algorithms for this problem in classes of graphs having relatively simple modular subgraphs. In particular, a linear time algorithm for determining the Grundy total domination number of $P_4$-tidy graphs is presented. Finally, we present a realization result, by exhibiting a family of graphs $G_k$ such that $\gamma_{tgr}(G_k) = k$ for every $k \in \mathbb{Z}^+ \setminus \{1, 3\}$, and showing that there are no graphs $G$ with $\gamma_{tgr}(G) \in \{1, 3\}$. 
On the complexity of $k$-tuple total and total \{$k\}$-domination in graphs

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Keywords: total \{$k\}$-domination problem, $k$-tuple total domination problem

For a graph $G$ with vertex set $V(G)$, a function $f : V(G) \mapsto \{0,1\}$ is a total dominating function of $G$ if $f(N(v)) \geq 1$ for all $v \in V(G)$, where $N(v)$ is the open neighborhood of the vertex $v$. The total domination problem consists in finding a minimum weight total dominating function of a given graph. We consider two variations of total domination introduced by M. Henning and A. Kazemi (2010), and N. Li and X. Hou (2009), respectively. Let $k$ be a nonnegative integer. A function $f : V(G) \mapsto \{0,1\}(\{0,1,\ldots,k\})$ is a $k$-tuple total dominating function (total \{$k\}$-dominating function) of $G$ if $f(N(v)) \geq k$ for all $v \in V(G)$. As usual, these definitions lead to the study of the corresponding decision problems for a nonnegative fixed integer $k$, namely $k$-DOM-T and \{$k\}$-DOM-T.

Concerning computational complexity results of the above problems, it is known that both of them are NP-complete on general graphs. The results up to now show that their computational complexity coincide in several classes of instances. In this work, we study those classes of instances whose complexity is already solved for $k$-tuple total domination with the purpose not only of solving them for total \{$k\}$-domination, but also of getting close to a proof or disproof of the existence of a polynomial equivalence between $k$-DOM-T and \{$k\}$-DOM-T (for fixed $k$). We prove that both problems are polynomial time solvable for bounded clique-width graphs and find the exact values of the corresponding parameters for cycles and complete multipartite graphs. Regarding NP-completeness results, we deal with split graphs and bipartite planar graphs.
Edge Coloring in Split-Comparability Graphs

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Keywords: edge coloring, chromatic index, split graphs, comparability graph

Let $G = (V, E)$ be a simple and undirected graph with vertex set $V$ and edge set $E$. A $k$-edge-coloring is an assignment of at most $k$ different colors to the edges of $G$ such that any two adjacent edges receive different colors. The chromatic index of a graph $G$, denoted by $\chi'(G)$, is the smallest $k$ such that $G$ has a $k$-edge-coloring. Let $\Delta(G)$ be the maximum degree of $G$. The lower bound for $\chi'(G)$ is $\Delta(G)$. In 1964, Vizing proved that $\chi'(G) \leq \Delta(G) + 1$. The Classification Problem is to determine if $\chi'(G) = \Delta(G)$ (Class 1) or $\chi'(G) = \Delta(G) + 1$ (Class 2). The Classification Problem is $\mathcal{NP}$-Complete.

A graph $G$ is overfull if $|V|$ is odd and $|E| > \frac{(|V|-1)}{2}\Delta(G)$. Every overfull graph is Class 2. The closed neighborhood of a vertex $v$ is the set $N[v] = N(v) \cup \{v\}$, where $N(v)$ denotes the set of vertices adjacent to $v$. A graph $G$ is neighborhood-overfull if for some vertex $v$ with degree $\Delta(G)$, $G[N[v]]$ is overfull. Every neighborhood-overfull graph is Class 2.

A clique is a set of pairwise adjacent vertices. A stable set is a set of pairwise non-adjacent vertices. A graph $G$ is a split graph if its vertices can be partitioned into a clique $Q$ and a stable set $S$. A graph $G$ is a comparability graph if it admits a transitive orientation of its edges. A split-comparability graph is a graph that is simultaneously split and comparability. We prove the following theorem:

**Theorem.** A split-comparability graph $G$ is Class 2 if and only if $G$ is neighborhood-overfull.

We conclude from this theorem and a result of Padberg and Rao from 1982 that the Classification Problem can be solved in polynomial time for split-comparability graphs.*

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On Maximum Colored Cuts in Edge Colored Graphs

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Keywords: Maximum cut. Edge colored graphs. Algorithms complexity.

Let $G^c = (V, E, c)$ be a simple connected graph $G = (V, E)$ with an edge coloring $c$, not necessarily proper. We study the problem of finding an edge cut using the maximum number of colors of $c$, which we call MAX-COLORED-CUT. We also consider the problem of finding an edge cut using all colors of $c$, which is a particular case of MAX-COLORED-CUT called COLORFUL CUT. In this work we prove that both problems are NP-complete even for complete graphs and that MAX-COLORED-CUT admits a polynomial 2-approximation algorithm. In addition, we present some polynomial and fixed parameter tractable cases.

References


Total Coloring and AVD Total Coloring in Complete Tripartite Graphs

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Keywords: total coloring, AVD total coloring, tripartite graphs

An r-partite graph is balanced if its vertex set admits a partition into stable sets with the same cardinality, and it is complete if the vertices in distinct sets of the partition are adjacent. A total-coloring of a graph $G$ is an assignment of colors to its vertices and edges such that adjacent elements receive distinct colors. The total chromatic number of a graph $G$, $\chi''(G)$, is the minimum number of colors needed for a total-coloring of $G$. A graph $G$ is type 1 when $\chi''(G) = \Delta(G) + 1$ and it is type 2 when $\chi''(G) = \Delta(G) + 2$. To decide if $G$ is type 1 is an NP-complete problem. Rosenfeld proved that every complete tripartite graph $G$ has $\chi''(G) \leq \Delta(G) + 2$. Bermond determined the total chromatic number of balanced complete r-partite graphs. Chew and Yap proved that every complete r-partite graph of odd order is type 1. In this work, we prove that complete tripartite graphs are all type 1.

Given a total coloring of a graph $G$, the color-set of a vertex $v$ is the set with the color of $v$ and the colors of the edges incident to it. An adjacent vertex distinguishing total coloring (AVD total coloring) is a total coloring such that the color-sets of adjacent vertices are distinct. The AVD total chromatic number of a graph $G$, denoted by $\chi''_a(G)$, is the minimum number of colors needed for an AVD total coloring of $G$. In 2014, Luiz conjectured that $\chi''_a(G) = \Delta(G) + 2$ for complete tripartite graphs with a partition $[A, B, C]$ such that $|B| = |C| \leq |A| < 2|B|$. In 2015, Luiz et al. conjectured that complete equipartite graphs with odd order have $\chi''_a(G) = \Delta(G) + 2$. In this work we prove that if $G$ is a tripartite graph, then $\chi''_a(G) = \Delta(G) + 2$ if and only if $G$ has adjacent vertices with maximum degree, otherwise $\chi''_a(G) = \Delta(G) + 1$. This result proves the conjecture of Luiz about complete tripartite graphs and it is an evidence for his conjecture for complete equipartite graphs.

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Caracterización espectral de la composición de ciertos grafos que poseen subgrafos inducidos completos por niveles

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Keywords: Autovalores, grafo Bethe generalizado.

Sea $\mathcal{B}$ un grafo con raíz y de $h$ niveles, tal que vértices en un mismo nivel tienen el mismo grado y, en uno o más niveles, hijos con un mismo padre forman un grafo completo.

En este trabajo obtenemos los autovalores de la matriz de adyacencia, Laplaciana y Laplaciana sin signo de tales grafos, a través de matrices tridiagonales simétricas de orden menor que el orden del grafo. En particular, a través de la unión de los espectros de matrices de orden menor o igual a $h$.

Finalmente, mostraremos resultados similares para grafos que resultan de la composición de $r$ de estos grafos adjuntando sus respectivas raíces a los vértices de un grafo $\mathcal{G}$.

References


Critical Ideals of Digraphs

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Keywords: Critical Ideals, Critical Group, Digraphs

Considering the Laplacian matrix of a digraph $D$ as a linear operator on $\mathbb{Z}^n$, the critical group $K(D)$ of $D$ is the torsion part of the cokernel of $L(D)^t$. The critical group has been studied intensively over the last 30 years on several contexts; namely sanpile groups.

We study a generalization of the critical group called critical ideals, which are defined from the determinantal ideals of the generalized Laplacian matrix. Critical ideals behave well under several operations like replication and duplication of vertices.

It is known that the only graph with one trivial critical ideal is the complete graph. We characterize the family of digraphs with one trivial critical ideals, and we also find a set of 17 digraphs that are minimal forbidden, under induced subdigraphs, for this family. Finally, we give a characterization of the digraphs whose critical group has one invariant factor equal to 1.
\(\mathcal{D}\)-integral and \(\mathcal{D}^Q\)-integral graphs

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**Keywords:** Complete Split Graph, \(M\)-characteristic polynomial, \(M\)-integral graph, distance Laplacian matrix, distance signless Laplacian matrix.

For a connected graph \(G\), we denote by \(D(G)\) the distance matrix of \(G\) and by \(T(G)\), the transmission matrix of \(G\), the diagonal matrix of the row sums of \(D(G)\). The matrices \(D^L(G) = T(G) - D(G)\) and \(D^Q(G) = T(G) + D(G)\) are called the distance Laplacian and the distance signless Laplacian of \(G\), respectively.

Among the aspects investigated in Spectral Graph Theory, one important problem is to characterize graphs for which all the eigenvalues of a matrix associated to them are integers. In a general way, if \(M\) is a real symmetric matrix associated to a graph \(G\), we say that \(G\) is \(M\)-integral when all the eigenvalues of \(M\) are integer numbers.

In this work we discuss \(M\)-integrality for some special classes of graphs, where \(M = D^L(G)\) or \(M = D^Q(G)\). In particular, we consider the classes of complete split graphs, multiple complete split-like graphs, extended complete split-like graphs and multiple extended complete split-like graphs, giving conditions for the \(M\)-integrality in each of them. Those classes were also considered to discuss the integrality for other matrices. In the case of \(D^L\)-integrality, we present a general characterization concerning them. For \(D^Q\)-integrality, besides determining the \(D^Q\)-characteristic polynomial of special graphs we construct infinite families of \(D^Q\)-integral graphs.
On graphs with linear convergence

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Keywords: clique, linear, convergence

Iterated clique graphs, whether they diverge or converge, tend to have an explosive behaviour in the first few steps. That is why in [1] (2002) the authors showed for the first time a class of graphs with linear divergence with respect to the number of vertices. In [2] (2013), evidence is shown that the topology of a graph is related with the $k$-behaviour. In this paper we will show two families of graphs, each of them closed and with linear convergence with respect to the number of vertices. In fact, the number of vertices is lowered by $m$ with $m \geq 4$ for the first family and $m \geq 3$ for the second family, leaving the question of whether it exists or not a family of graphs that converge to a non-trivial graph decreasing the number of vertices by 1 or 2 in each step. The first family behaves like cylinders, and the second class like Möbius strips.

References


On Strong Graph Bundles and Clique Graphs

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\textit{Keywords: fiber bundles, strong product, clique graphs}

In topology, a \textit{fiber bundle} is a space which locally looks like a product of spaces. This concept has proved to be very important in many fields of mathematics including algebraic geometry, differential geometry and differential topology. Also, fiber bundles play a central role in general relativity. Thus the importance of fiber bundles in mathematics and physics is difficult to overstate.

The analogues of fiber bundles in graph theory, i.e., \textit{graph bundles}, were introduced by Pisanski and Vrabec in 1982. As you would expect, a graph bundle is a graph which locally looks like a product of graphs... but a “product of graphs” may refer to several different things: strong products, Cartesian products, tensor products, etc. All of these products have been studied in the literature. The product best suited for our purposes is the strong product, and hence we studied the \textit{strong graph bundles}, which are graphs that locally look like a strong product of graphs. Strong graph bundles generalize both triangular covering graphs and strong product of graphs.

As an application of our study, we show that the clique operator (which transforms a graph into the intersection graph of its maximal cliques) preserves strong graph bundles. This introduces a new technique for clique divergence: a strong graph bundle is clique divergent if and only if either its \textit{base graph} or its \textit{fiber graph} is clique divergent.
Let $H$ be a graph. The clique graph $K(H)$ of $H$ is defined as having the cliques of $H$ as vertices and two cliques $C_i$ and $C_j$ being adjacent in $K(H)$ if and only if $C_i \cap C_j \neq \emptyset$. The graph $H$ is said to be clique critical if $K(H) \neq K(H - x)$ for all $x \in V(H)$, where $H - x$ is the graph obtained from $H$ by removing the vertex $x$ and all the edges incident on it [1]. If $H$ is a clique critical graph, then we say that $H$ is a critical generator of $G = K(H)$. It is known that any clique graph $G$ admits a finite number of critical generators. One of the main problems concerning clique critical graphs is to find which graphs have a unique critical generator. Escalante and Toft [1] showed that if $G$ is triangle free then $G$ has a unique critical generator. In [2], the authors characterize the diamond free-graphs with unique critical generator.

In this work, we present necessary and sufficient conditions so that a $K_4$-free clique graph has a unique critical generator and so that a $K_5$-free clique graph with each triangle contained in at most one $K_4$ has a unique critical generator.

References


Equitable total colouring of Loupekine Snarks and its products

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Keywords: total colouring, equitable total colouring, snark

A $k$-total-colouring of a graph $G$ is an assignment of $k$ colours to the vertices and edges of $G$ such that adjacent and incident elements have different colours, and the total chromatic number of a graph $G$ is the least integer for which a graph has such a coloring. Due to the well-known Total Colouring Conjecture, the total chromatic number of a cubic graph is either 4 (in which case the graph is called Type 1) or 5 (Type 2). A total colouring is equitable if the number of elements colored with each color differs by at most one; the least integer for which a graph has such a coloring is called its equitable total chromatic number.

Wang proved in 2012 that the equitable total chromatic number of a cubic graph is either 4 (here called equitable Type 1) or 5 (equitable Type 2). As the equitable total chromatic number of a graph cannot be smaller than its total chromatic number, it follows that every cubic graph with no 4-total-colouring has equitable total chromatic number 5. Nevertheless, there are infinitely many Type 1 cubic graphs that admit no equitable 4-total colouring.

Motivated by the question proposed by Dantas et al. (2016) about the existence of a Type 1 cubic graph with girth at least 5 and equitable total chromatic number 5, we prove that every member of an infinite subfamily of Loupekine snarks (girth 5) has an equitable 4-total-colouring. Sasaki showed in 2013 that the dot product of two Type 1 cubic graphs may be Type 2, but little is known about snarks. We determine equitable 4-total-colourings for graphs of infinite subfamilies obtained by the dot product of the Loupekine snarks and the Flower, Goldberg and Blanuša snarks. We show that the dot product of two equitable Type 1 snarks may be an equitable Type 2 snark.

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Equitable total coloring of complete $r$-partite graphs $^a$

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Keywords: equitable total coloring; complete $r$-partite graph; algorithm

A total coloring $C : V \cup E \to S$ of a graph $G = (V, E)$, where $S$ is a set of $k$ colors, is a $k$-equitable total coloring if the difference between the cardinalities of any two color classes is either 0 or 1. The equitable total chromatic number $\chi''_e$ of $G$ is the least $k$ for which $G$ has a $k$-equitable total coloring. A graph is complete $r$-partite if there exists a partition of its vertex set into $r$ independent sets in a way that every two vertices in different sets of the partition are adjacent. We denote by $K_{p_1,p_2,\ldots,p_r}$ a complete $r$-partite graph having $p_i$ vertices in each independent set; and by $K_{r\times p}$ if each $p_i = p$, $i = 1, \ldots, r$. In 1974, Bermond investigated the total coloring of all complete $r$-partite graphs $[1]$. Later, Wang $[2]$ conjectured that the equitable total chromatic number of any graph is at most $\Delta + 2$, where $\Delta$ is the maximum degree of a graph. Motivated by these results, we verify this conjecture for infinite families of complete $r$-partite graphs, by showing algorithms to determine the equitable total chromatic number of the following graphs:

1. $K_{2\times p}$, has $\chi''_e = \Delta + 2$;
2. $K_{p_1,p_2}$, with $p_1 \neq p_2$, has $\chi''_e = \Delta + 1$;
3. $K_{r\times p}$, with $r$ even and $p$ odd, has $\chi''_e = \Delta + 2$;
4. $K_{r\times p}$, with $r$ odd and $p$ even, has $\chi''_e = \Delta + 1$.

References


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On the parameterized complexity of finding all the cliques of a graph.

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Keywords: Bron-Kerbosch algorithm, parameterized time complexity.

A clique of a graph $G$ is a maximal complete subgraph of $G$. It is well known that every graph with $n$ vertices has at most $3^{\frac{n}{3}}$ cliques. Therefore, in the worst-case scenario, any algorithm that enumerates all the cliques of a graph needs exponential time.

In this talk we will review one of the most widely used algorithms for finding cliques in a graph: the Bron-Kerbosh algorithm. We will examine variants of this algorithm as well as the time complexity found by Tomita et al. for this algorithm: $O(\omega 3^{\frac{n}{3}})$ for graphs with $n$ vertices and with clique number $\omega$.

We will also share our progress in trying to find a better upper bound for the parameterized time complexity of the Bron-Kerbosh algorithm than the one found by Tomita et al.. For a graph with $n$ vertices, $m$ edges, $\mu$ cliques and clique number $\omega$, our current conjecture is that the time complexity of the Bron-Kerbosh algorithm is $O(n^2 + nm + \omega \mu)$. If the algorithm does not print every vertex of each clique that it founds, then our conjecture is that the time complexity is $O(n^2 + nm + \mu)$.
On the clique behavior’s undecidability for finitely presented graphs

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Keywords: Undecidability, Clique graph, Clique behavior

Let $G$ be a graph. A clique of $G$ is a maximal complete subgraph of $G$. The intersection graph of its cliques is the clique graph $K(G)$ of $G$. The iterated clique graphs are defined recursively by $K^0(G) = G$ and $K^{n+1}(G) = K(K^n(G))$. We say that $G$ is $K$-convergent if $K^n(G) \cong K^m(G)$ for some $n \neq m$; otherwise, we say that $G$ is $K$-divergent. The clique behavior problem consists in determining whether a given graph is $K$-convergent or not.

We will show that the reachability problem for (certain) infinite digraphs reduces to the clique behavior problem of finitely presented (but infinite) graphs. Now, the reachability problem is easy to solve for finite digraphs, but the problem becomes (algorithmically) undecidable when the digraphs considered are infinite even under very strict additional conditions like the following: finitely presented, countable, locally finite, connected, degree at most 2, etc. This reduction allows us to prove that the clique behavior problem is also undecidable for the family of finitely presented graphs used in the proof (and any superfamily of it).

We will also present our advances in extending this result to the case of finite graphs. In this endeavor, we have been simulating electronic circuits and, so far, we have obtained already the analogues of cables, Y - splices, OR gates, AND gates, oscillators and more. We only need to find the NOT gate so we can simulate an entire digital computer in terms of graphs and the dynamics of the clique operator. This would arguably solve in the positive a question posed by João Meidanis in 2001:

*Does $K$ have the computational power of Turing machines? In other words, is it possible to code an arbitrary algorithm and its input as a graph, and simulate the algorithm steps by iterated application of $K$? (http://www.ic.unicamp.br/~meidanis/research/clique/**/)*
On unit $d$–interval graphs

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Keywords: unit interval graphs, forbidden subgraphs, proper interval graphs

A $d$–interval is the union of $d$ disjoint intervals on the real line. A $d$–interval graph is the intersection graph of a family of $d$–intervals. Some subclasses of $d$–intervals can be built by restricting the $d$ disjoint intervals lengths as follows: A $d$–interval is balanced if the $d$ disjoint intervals have the same length. A $d$–interval is unit if the $d$ disjoint intervals all have unit length.

The problem of recognizing if a graph $G$ is a unit interval graph leads to well-known efficient results. For instance, $G$ is unit interval if and only if it is (i) $\{\text{net, tent, } C_k\}$–free, for all $k \geq 4$ and (ii) $K_{1,3}$–free. The item (i) characterizes the class of interval graphs. Therefore, if we assume that $G$ is an interval graph, the characterization is simple: $G$ is unit if and only if $G$ is $K_{1,3}$–free. As shown at the table below, the problem of characterizing if a graph $G$ is unit $d$–interval is still open.

Similarly to the characterization of unit interval graphs from a given interval graph, we can study if this open problem admits a simple and direct characterization when starting with an interval graph $G$. We present a simple and efficient algorithm which answers positively this open problem, and its characterization is a generalization of the case $d = 1$.

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Current complexities of recognizing variants of $d$–interval graphs.
Characterizing probe unit interval graphs within the class of interval graphs

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**Keywords:** interval graphs, probe interval graphs, probe unit interval graphs, unit interval graphs

A graph \(G\) is an *interval graph* if it is the intersection graph of a family \(S\) of closed intervals in the real line. Such a family \(S\) is called an *interval model* of \(G\). A *unit interval graph* is an interval graph admitting an interval model with all its intervals having the same length. Interval graphs have a well-known characterization by forbidden induced subgraphs \([1]\). Roberts proved that an interval graph is a unit interval graph if and only if it does not contain a claw (the claw \(K_{1,3}\)) as an induced subgraph \([2]\).

A graph is *probe unit interval* if its vertices can be partitioned into two sets: a set of *probe vertices* and a set of *nonprobe vertices*, so that the set of nonprobe vertices is a stable set and it is possible to obtain a unit interval graph by adding edges with both endpoints in the set of nonprobe vertices. In view of Roberts’ characterization, a natural question comes up: What is the family of minimal forbidden induced interval graphs for the class of probe unit interval graphs? In this work we answer this question by presenting a characterization by minimal forbidden induced subgraphs for the class of probe unit interval graphs, within the class of interval graphs.

**References**


A structural characterization of minimal completions from interval graphs to proper interval graphs

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**Keywords:** completion, minimal, interval, proper interval, structural characterization

A \(\Pi\)-completion \(H = (V, E \cup F)\) of a graph \(G = (V, E)\) is called minimal if for every proper subset \(F'\) of \(F\) the graph \(H' = (V, E \cup F')\) does not belong to the class \(\Pi\), and its minimum if for every \(\Pi\)-completion \(H' = (V, E \cup F')\) the size of \(F\) is smaller than or equal to the size of \(F'\). It is clear that every minimum \(\Pi\)-completion is minimal, but the reciprocal is not true.

The problem of calculating a minimum completion from an arbitrary graph to a specific one is very important and quite studied since it has applications in various areas like molecular biology, computational algebra, and more specifically in areas that involve modelling based on graphs where the missing edges represent lack of data, such as in clustering data problems. Unfortunately, minimum arbitrary graph completions to most graph classes are NP-hard problems. For this reason, current investigations are focused on finding minimal completions for arbitrary graphs to specific classes in the most efficient possible way from the computational point of view.

In this work, we find a structural characterization for minimal completions in the case where the initial graph \(G\) belongs to the class of interval graphs, and the completion is a graph \(H\) in the class of proper interval graphs. This characterization allowed us to find a polynomial time recognition algorithm for minimal completions in this particular case.
Sobre Grafos ORTH[3,2,3]

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Keywords: Grafos de Interseção de Subárvores, Grafos de Tolerância

Uma \((h, s, t)\)-representação de um grafo \(G\) [1] consiste em representar \(G\) como grafo de interseção de subárvores de grau máximo \(s\) de uma árvore hospedeira \(T\) de grau máximo \(h\), tal que dois vértices são adjacentes em \(G\) se e somente se as suas subárvores tem interseção de tamanho pelo menos \(t\), ou seja, compartilham no mínimo \(t\) vértices. Denotamos por \([h, s, t]\) a classe de grafos que admitem uma \((h, s, t)\)-representação. Um grafo \(G\) é ORTH\([h, s, t]\) se possui uma \((h, s, t)\)-representação ortodoxa. Isto é, uma \((h, s, t)\)-representação de \(G\) na qual cada uma das subárvores que representam os vértices de \(G\) são tais que suas folhas também são folhas da árvore hospedeira \(T\), e duas subárvores compartilham uma folha se e somente se os vértices correspondentes são adjacentes em \(G\).

Em [2], estudou-se os \(k\)-EPT grafos que equivalem aos \([\infty, 2, k+1]\). Neste trabalho os autores deixam como problemas em aberto caracterizar a classe ORTH\([3, 2, k+1]\) e verific se esta classe é equivalente a ORTH\([\infty, 2, k+1]\). Em [3], caracterizou-se através de subgrafos induzidos proibidos as classes \([3,2,1]\) e \([3,2,2]\). Em nosso trabalho, estudamos os grafos da classe ORTH\([3,2,3]\). Mostramos que se \(G\) é ORTH\([3,2,3]\) então admite uma cobertura de arestas por cliques onde cada um de seus vértices pertence a no máximo duas cliques. Exibimos grafos linha que não são grafos ORTH\([3,2,3]\), provando que os grafos ORTH\([3,2,3]\) estão propriamente contidos na classe dos grafos linha de multigráficos.

Bounds on the number of tessellations in graphs

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Keywords: graph tessellations, clique graphs, quantum walks, staggered model.

Given a graph $G$, a tessellation is a maximal set $T_i$ of disjoint cliques of $G$ that cover all the vertices of $G$. A graph $G$ is $T$-tessellable if $T$ is the minimum number for which there exist tessellations $T_1, \ldots, T_T$ such that $T_1 \cup \ldots \cup T_T$ covers all the edges of $G$. The Staggered Model (SM) yields a discrete-time quantum walk on a graph without obligating a coin operator [2]. From the tessellations of the graph, the SM defines orthogonal reflexive operators which are then composed in order to obtain the evolution operator.

Portugal [1] showed that $G$ is 2-tessellable if and only if its clique graph $K(G)$ is 2-colorable. On the other hand, the sufficient condition is not generalized for $T \geq 3$. Moreover, the characterization of the number of tessellations is still an open problem. We prove that the number of tessellations satisfies the inequality $\left\lceil \frac{\chi(K(G))}{2} \right\rceil \leq T(G) \leq \chi(K(G))$ for every graph $G$. We also prove that, for wheel graphs, $T(G) = \left\lceil \frac{\chi(K(G))}{2} \right\rceil$, while for graphs where every maximal clique shares just a cut vertex with all others, $T(G) = \chi(K(G))$.

References


(3,4,6)-Fullerene Graphs, Combinatorial Curvature Concept, Bipartite Edge Frustration and Maximum Independent Set

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Keywords: Bipartite Edge Frustration, Combinatorial Curvature, (3,4,6)-Fullerene Graphs.

A (3,4,6)-Fullerene Graph is a cubic bridgeless plane graph with all faces of sizes 3, 4 or 6. Let \(G = (V, E)\) be a graph and \(e = uv \in E\). Došlic and Vukičević² defined edge \(e\) as frustrated with respect to a given bipartition \(V = (V₁, V₂)\) of \(V\) if \(u\) and \(v\) belong to the same class of the bipartition. The Bipartite Edge Frustration Problem consists of determining the smallest number of edges that have to be deleted from \(E\) in order to obtain a bipartite spanning subgraph of \(G\) and this smallest number is denoted by \(\tau_{\text{odd}}(G)\). We investigate the Bipartite Edge Frustration and the Maximum Independent Set Problems on (3, 4, 6)-fullerene graphs. Both results are given by Theorem 1.

**Teorema 1** If \(G = (V, E)\) is a (3,4,6)-fullerene graph on \(n = |V|\) vertices, then \(\tau_{\text{odd}}(G) \leq \sqrt{\frac{4}{3}n}\). Furthermore, \(\alpha(G) \geq n/2 - \sqrt{n}/3\). Equality holds if and only if \(n = 12k²\), for some \(k \in \mathbb{N}\), and \(\text{Aut}(G) \cong T₄\).

In order to prove Theorem 1, we investigate the dual graph of a (3,4,6)-fullerene graph and we use the Combinatorial Curvature for planar graphs.

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The Petersen Graph Synchronizes

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Keywords: Kuramoto Model, synchronization of oscillators, global stability.

A graph $G$ synchronizes if almost all the solutions of the Kuramoto homogeneous model goes to a consensus, where, the Kuramoto homogeneous model is the system of differential equations over $\mathbb{R}^{V(G)}$ given by

$$\dot{\theta}_i = \sum_{ij \in E(G)} \sin(\theta_j - \theta_i) \quad i \in V(G)$$

and a consensus is any equilibrium of the form $\theta_i(t) = \theta^*$ for all $i \in V$ and some $\theta^* \in \mathbb{R}$.

In this work we prove that the Petersen graph synchronizes. The proof uses combinatorial properties of the Petersen graph in order to find a superset of the non consensus equilibria where the Hessian of certain Lyapunov function has a negative eigenvalue. Since finding the eigenvalue is a difficult task, we evaluate the quadratic form in the complex vector associated to the positions of the phasors $e^{\theta_i I}$ with $i \in V$, where $I$ is the imaginary unit, proving that these evaluations are negative everywhere.

Previous results include the synchronization of graph’s families like trees and complete graphs (Monzón-Paganini 2006), wheels (C.-Monzón-Robledo 2010) and complete $k$-partite graphs (C.-Monzón 2009). Besides, more general results include the reduction of the property to the blocks of the graph (C.-Monzón 2007), the fact that every connected graph is the induced subgraph of a synchronized one, and that any graph with at least one cycle is homeomorphic to a non synchronized one (C.-Monzón 2008), and the synchronization of any connected graph with minimum degree greater than 0.9395($n - 1$) (Taylor 2012). It is worth to say that there is still no combinatorial characterization of synchronizing graphs.
On fractional graph and hypergraph isomorphism and its applications

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Keywords: fractional graph theory, packing, matching, hypergraphs.

Fractional isomorphism of graphs is the linear relaxation of an integer programming formulation of graph isomorphism [1]. It preserves some invariants of graphs, but it does not preserve others like connectivity, clique number, chromatic number, and matching number.

In this work, we extend the concept of fractional isomorphism of graphs to hypergraphs, and give alternative characterizations, analogous to those that are known for graphs. With this new concept we prove that the fractional packing and covering numbers on hypergraphs are invariant under fractional hypergraph isomorphism, and as a consequence we prove that fractional matching is invariant under fractional graph isomorphism.

We also study the validity of an analogous to Whitney’s theorem for line graphs [2] applied to fractional isomorphism, thus obtaining partial results in this direction.

References


On the geodesic Carathéodory number for the cartesian product of graphs

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Keywords: Graph, Carathéodory Number, Geodesic Convexity, Path Convexity, Cartesian Product

Let $G$ be a graph given by a set of vertices $V(G)$ and a set of edges $E(G)$. A set $C$ of subsets of $V(G)$ is a convexity if $\emptyset, V(G) \in C$ and $C$ is closed under intersections. The elements of $C$ are called convex sets. Given a set $S \subset V$, the convex hull of $S$ is the smallest convex set $H(S) \in C$ such that $S \subseteq H(S)$.

Several convexities are defined by a set $\mathcal{P}$ of paths in a graph. In this scenario, a subset $C \in V(G)$ is convex when for all $x, y \in C$, if $p \in \mathcal{P}$ is a path between $x$ and $y$, then all vertices in $p$ are also in $C$. The geodesic convexity is obtained when $\mathcal{P}$ is the set of all the shortest paths.

A subset $S \subset V(G)$ is called a Carathéodory set when $\partial H(S) = H(S) \setminus \bigcup_{s \in S} H(S \setminus \{s\})$ is nonempty. The Carathéodory number is the maximum cardinality of a Carathéodory set [1].

In this work, regarding the geodesic convexity, we show that the Carathéodory number is unlimited for cartesian products, and we also characterize the Carathéodory number for the cartesian product of basic graphs ($P_n$ and $K_n$).

References

Sobre o número de Helly na convexidade $P_3$

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Keywords: Convexidade, convexidade $P_3$, Número de Helly

Dado um grafo $G$, um conjunto de vértices $S$, tal que $S \subseteq V(G)$, é dito um $P_3$-convexo se todos os vértices com ao menos dois vizinhos em $S$ pertencem a $S$.

Existem alguns parâmetros associados a convexidades em grafos bastante estudados como posto, também conhecido como $rank$, número de Radon e número de Carathéodory.

No contexto da convexidade $P_3$ estudamos o parâmetro conhecido como número de Helly, definido como o menor número inteiro $k$ para o qual toda subfamília $k$-intersectante $C$, composta por subconjuntos não vazios de $V(G)$ (onde cada subconjunto de $V(G)$ em $C$ é um $P_3$-convexo) possui um vértice comum a todos os conjuntos da família $C$. Denotamos o parâmetro por $h_{P_3}(G)$, ou simplesmente $h(G)$, caso não haja ambiguidade.

Apresentamos um teorema que permite determinar o valor do parâmetro de modo direto para grafos que contenham um vértice universal. Tais grafos atendem à propriedade de Helly, ou seja, $h(G) = 2$.

Determinamos também o número de Helly nesta convexidade para algumas classes de grafos mais simples como caminhos, ciclos, grafos completos, grafos $k$-partidos completos e grafos de limiar, também conhecidos como grafos threshold.

Para grafos caminhos e ciclos, o parâmetro varia de acordo com o tamanho do grafo; já para grafos completos, $k$-partidos completos e grafos de limiar mostramos que estas classes de grafos atendem à propriedade de Helly nesta convexidade.

Finalmente, são descritos alguns limitantes inferiores para o número de Helly de um grafo na convexidade $P_3$. 
Convexidade de Precessão em Digrafos

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Palavras-chave: Convexidade de Precessão, Grafos Dirigidos, Algoritmos

Seja um grafo dirigido $G(V,E)$ e a função $f : V(G) \to \{0, 1, 3, \ldots, j\}$. Um conjunto $S \subset V(G)$ é convexo de precessão se $\forall v \notin S$ possui $|N^-(v) \cap S| < f(v)$. Definimos o número de convexidade de precessão de $G$, $c_p(G)$, como a cardinalidade do maior conjunto convexo de precessão próprio de $G$. O intervalo de precessão de um conjunto $S \subset V(G)$, é definido por $I_p(S) = S \cup \{v \in V(G) : |N^-(v) \cap S| \geq f(v)\}$. Quando $I_p(S) = V(G)$, $S$ é chamado de conjunto de precessão. O número de precessão de $G$, $p_p(G)$, é a cardinalidade do menor conjunto de precessão. Dado um subconjunto $S$ de $V(G)$, $I_p^+(S) = S_i$ onde $S_1 = I_p(S), S_2 = I_p(S_1), S_3 = I_p(S_2), \ldots, S_i = I_p(S_{i-1})$ e $S_i$ é um conjunto convexo de precessão. O conjunto $S$ é uma envoltória convexa de precessão. Quando $I_p^+(S) = V(G)$, então $S$ é uma envoltória de precessão de $G$. O número da envoltória de precessão, $h_p(G)$, é a cardinalidade da menor envoltória de precessão de $G$.

Em Centeno [1] é estudada a convexidade de precessão para grafos dirigidos acíclicos, onde são determinados intervalos para os números de convexidade de precessão e envoltória de precessão; trabalho este apresentado no LAWCG 2012. Ainda é mostrado que para grafos dirigidos acíclicos o número de precessão é um problema NP-Completo. Neste trabalho abordamos o problema para grafos dirigidos cíclicos. Construímos algoritmos polinomiais que respondem os três problemas para as classes de grafos dirigidos transitivos e 2-regulares, onde $\forall v \in V(G)$, $f(v) = k$, e $k > 1$. Ainda resolvemos o problema para grafos dirigidos quando a função $f$ é limitada.

Referência

Vertex intersection graphs of paths on a grid: a characterization within block graphs

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Keywords: Vertex intersection graphs, paths on a grid, forbidden induced subgraphs, block graphs.

A VPG representation of a graph $G$ is a collection of paths of the two-dimensional grid where the paths represent the vertices of $G$ in such a way that two vertices of $G$ are adjacent in $G$ if and only if their corresponding paths share at least one vertex of the grid. A graph which has a VPG representation is called a VPG graph. In this work, we consider the subclass $B_0$-VPG.

A $B_0$-VPG representation of $G$ is a VPG representation in which each path in the representation is either a horizontal path or a vertical path on the grid. A graph is a $B_0$-VPG graph if it has a $B_0$-VPG representation.

Recognizing this class is an NP-complete problem, although there exists a polynomial time algorithm for recognizing chordal $B_0$-VPG graphs.

In this work, we present a minimal forbidden induced subgraph characterization of $B_0$-VPG graphs restricted to block graphs. As a byproduct, the proof of the main theorem provides an alternative certifying recognition and representation algorithm for $B_0$-VPG graphs in the class of block graphs.
Maximal Independent Sets in Simple Graphs

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Keywords: maximal independent set, counting, independent graph, clique graph, sun graph, wheel, helm, tadpole, lollipop.

The Independent Graph $\mathcal{I}(G)$ of a graph $G$ is the intersection graph of the family of its maximal independent sets (mis). In this work, we determine the number of mis and we characterize the independent graph and the clique graph of simple graphs that have a clique or a cycle as induced subgraphs. We use the results by Füredi who gave the number of mis of a path and a cycle. Prodinger and Tichy established the Fibonacci number (number of independent sets) of a cycle. Despite the graphs are similar, their independent graphs are very different.

The complete sun graph $S(K_n)$ consists of a clique $K_n$ with vertices $v_1, .., v_n$ and an independent set $R_n$ with vertices $r_1, .., r_n$ such that $r_i$ is adjacent to $v_i$ and $v_{i+1}$ (mod $n$). The chordless sun $S(C_n)$ has a chordless cycle $C_n$ instead of the clique $K_n$. A wheel graph $W_{1,n}$ has a single vertex $w$ connected to all the vertices of an $n$-cycle. A helm graph $H_{1,n}$ is obtained by attaching a pendant vertex to each vertex of the outer cycle of a wheel graph $W_{1,n}$. The lollipop graph $L_{m,n}$ is obtained by joining a complete graph $K_m$ to a path $P_n$ with a bridge. The tadpole graph $T_{m,n}$ has a chordless cycle $C_m$ connected to a chordless path $P_n$ by a bridge.

References
