

Equitable total coloring of complete r -partite graphs

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Introduction

- A k -total coloring is the assignment of k colors to the vertices and edges of a graph such that incident and adjacent elements receive different colors.

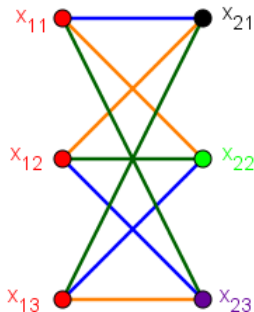


Figure 1: Total coloring of $K_{2 \times 3}$

Total Coloring Conjecture

The total chromatic number of any graph, denoted by χ'' , is such that $\Delta + 1 \leq \chi'' \leq \Delta + 2$, where Δ is the maximum degree of the graph.

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- If the difference between the cardinalities of any two color classes is either 0 or 1, the total coloring is said to be equitable.

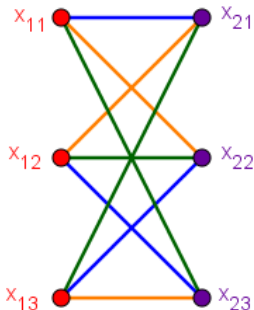


Figure 2: Equitable total coloring of $K_{2 \times 3}$

Equitable Total Coloring Conjecture

The equitable total chromatic number of any graph, denoted by χ''_e , is such that $\Delta + 1 \leq \chi''_e \leq \Delta + 2$.

Introduction

- A *complete r -partite graph* is a graph whose vertex set can be partitioned into r parts such that no two vertices within the same part are adjacent and in which there is an edge between any two vertices of different parts of the partition.
- If the cardinality of each part of the partition of the vertex set is p , the graph is said to be *p -balanced*.

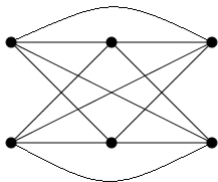


Figure 3: Complete 3-partite 2-balanced graph

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- Bermond (1974) established the total chromatic number of complete r -partite p -balanced graphs and Fu (1994) investigated the equitable total coloring of complete bipartite p -balanced graphs.
- In this work, we determine the equitable total chromatic number of the following graphs and establish algorithms to obtain equitable total colorings of such graphs:
 - 1 $K_{2 \times p}$, $\chi_e'' = \Delta + 2$;
 - 2 K_{p_1, p_2} , with $p_1 \neq p_2$, $\chi_e'' = \Delta + 1$;
 - 3 $K_{r \times p}$ with r even and p odd, $\chi_e'' = \Delta + 2$;

Equitable total coloring of $K_{2 \times p}$

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Matrix representation of an
equitable total coloring of $K_{2 \times 3}$
using 5 colors:

$$A_{X_1 X_2} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$X_1 \rightarrow 4$ and $X_2 \rightarrow 5$

● 1 ● 4

● 2 ● 5

● 3

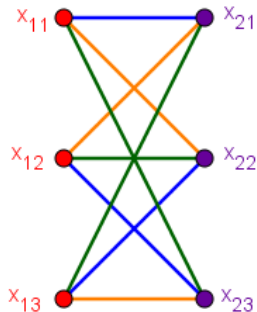


Figure 4: Equitable total coloring of $K_{2 \times 3}$

Equitable total coloring of K_{p_1, p_2}

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Matrix representation of an
equitable total coloring of $K_{3,4}$
using 5 colors:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix}$$

$X_1 \rightarrow 5$

● 1 ● 4

● 2 ● 5

● 3

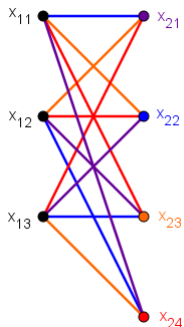


Figure 5: Equitable total coloring of $K_{3,4}$

Equitable total coloring of $K_{r \times p}$ with r even (≥ 4) and p odd

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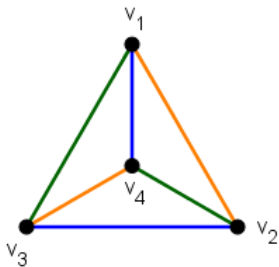


Figure 6: Disjoint matchings of K_4

Original edge-coloring matrices:

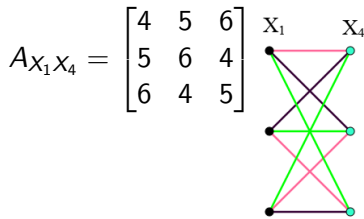
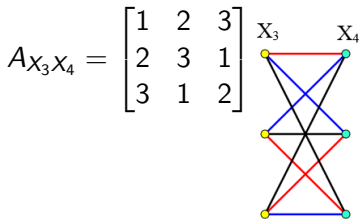
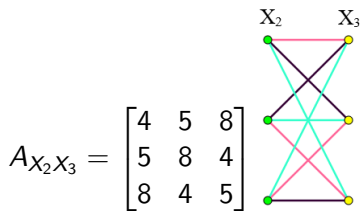
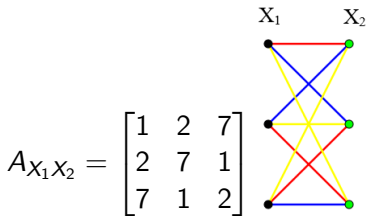
$$A_{X_1 X_2} = A_{X_3 X_4} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$A_{X_2 X_3} = A_{X_1 X_4} = \begin{bmatrix} 4 & 5 & 6 \\ 5 & 6 & 4 \\ 6 & 4 & 5 \end{bmatrix}$$

Equitable total coloring of $K_{r \times p}$ with r even (≥ 4) and p odd

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$$X_1 \rightarrow 3, X_2 \rightarrow 6, X_3 \rightarrow 7, X_4 \rightarrow 8$$

References

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A. da Silva,,
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Sasaki



W. F. Wang

Equitable total coloring of graphs with maximum degree 3.
Graphs Combin., 2002.



A. Sánchez-Arroyo

Determining the total colouring number is NP-hard.
Discrete Math, 1989.



H. L. Fu

Some results on equalized total coloring.
Congr. Numer., 1994.



M. Behzad

Graphs and their chromatic numbers.
Michigan State University, 1965.

References

Equitable
total coloring
of complete
 r -partite
graphs

A. da Silva,,
S. Dantas,, D.
Sasaki



V. G. Vizing

Some unsolved problems in graph theory.

Russian Math. Surveys, 1968.



S. Dantas and C. M. H. de Figueiredo and G. Mazzuocolo
and M. Preissmann and V. F. dos Santos and D. Sasaki

On equitable total chromatic number of cubic graphs.

Discrete Appl. Math, 2016



J. C. Bermond

Nombre chromatique total du graphe r -parti complet.

J. London Math. Soc., 1974