K_5-free Clique Graphs with each Triangle Contained in at most one K_4 with Unique Critical Generator

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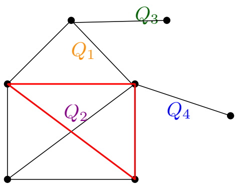
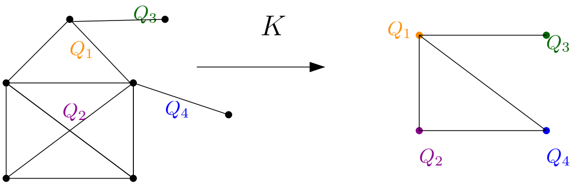
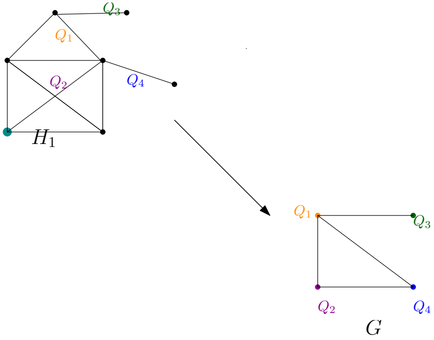
A complete set is a set of mutually adjacent vertices.
A clique is a maximal complete set (in the sense of set inclusion).



$\mathcal{C}(H)$ is the cliques family of H. The clique graph of H, $K(H)$, is the intersection graph of $\mathcal{C}(H)$.



A vertex x is superfluos if $K(H-x) = K(H)$. Otherwise it is critical.



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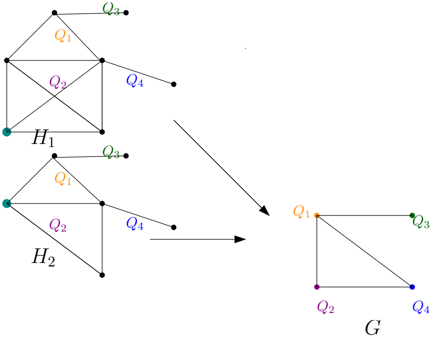

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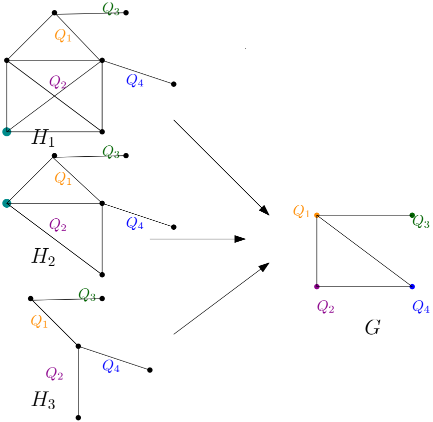
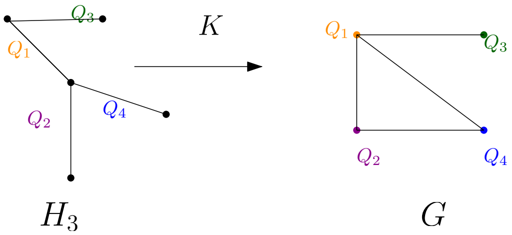
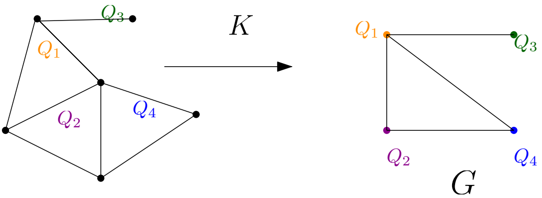


A graph H is critical if all its vertices are critical. In that case H is a critical generator of $K(H)$.



Is H_3 the only critical generator of G?



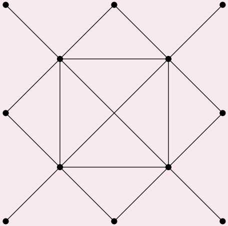
Let G be a graph and $K^{-1}(G)$ be the set of all graphs H such that $G = K(H)$

- A necessary and sufficient condition for $K^{-1}(G) \neq \emptyset$ is given by Robert and Spencer.
- Escalante and Toft show that the number of clique critical graphs in $K^{-1}(G)$ is always finite. They proposed the problem of characterizing the graphs with unique critical generator. They solved the problem in the family of K_3-free graphs.
- Chong-Keang and Yee-Hock solved the problem in the family of clique graphs that have all the edges in exactly one clique.

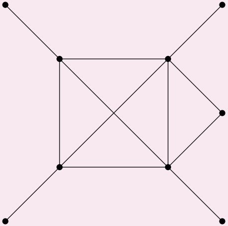
Theorem

Let G be a clique graph, K_5-free, such that every triangle is contained in at most one K_4.
Then G has an only one critical generator if and only if

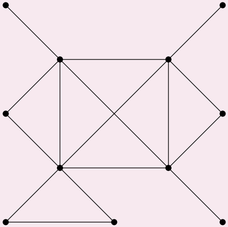
1. each K_4 has, at least, 2 multicliqual edges without a common vertex and;
2. for all $v \in V(G)$ at least one of the next statements holds:
   - v has degree 1,
   - $\{v\} = C_1 \cap C_2$ for some $C_1, C_2 \in \mathcal{C}(G)$,
   - $v \notin V(K_4)$, all opposite edge to v is multicliqual.



Satisfies the condition of the theorem



Doesn't satisfy



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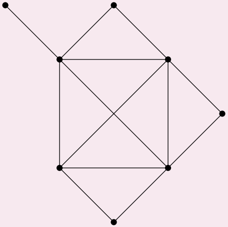

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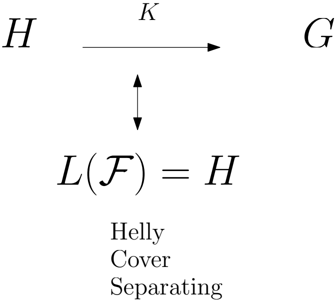
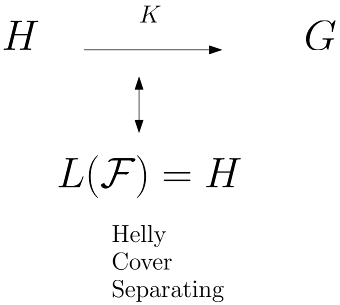


- <u>Helly</u> if every interesecting subfamily has a total intersection nonempty;
- <u>cover</u> if each edge of G belongs to some member of $\mathcal{F}$;
- <u>separating</u> if for each $v \in V(G)$ the intersection of all the members of $\mathcal{F}$ containing v is exactly $\{v\}$.



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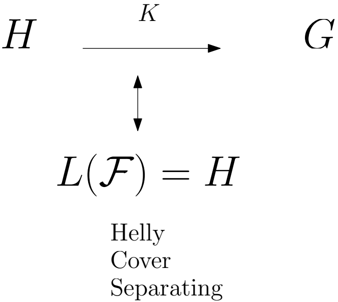

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$$H \xrightarrow{K} G$$

$\updownarrow$

$$L(\mathcal{F}) = H$$

Helly
Cover
Separating

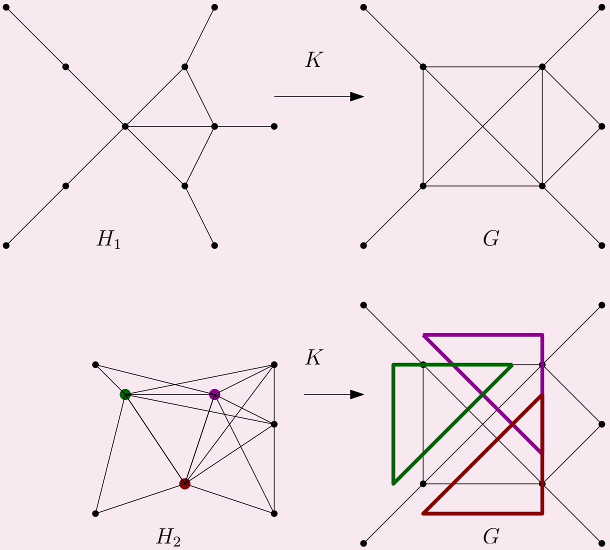
Helly if every interesecting subfamily has a total intersection nonempty;

$L(\mathcal{F})$ is a critical generator iff $\mathcal{F}$ is minimal (in the sense that $\mathcal{F} - F$ is not cover or is not separating).

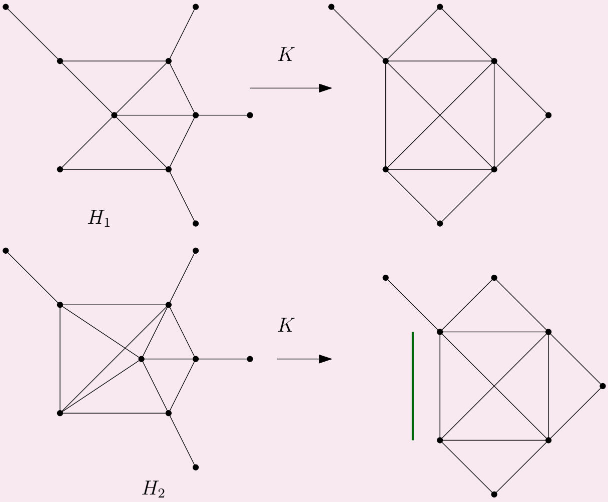
separating if for each $v \in V(G)$ the intersection of all the members of $\mathcal{F}$ containing v is exactly $\{v\}$.

The idea is to prove that $\mathcal{F} = \mathcal{C}(G) \cup \{\{v\} : v \text{ is not separated by } \mathcal{C}(G)\}$ is the unique minimal Helly, cover and separating family of G.

A graph that no satisfy the condition 1 of the Theorem



A graph that no satisfy the condition 2 of the Theorem



Corollary of the Theorem

Corollary

Let G be a clique graph, K_4-free. G has an only critical generator if and only if for all $v \in V(G)$ at least one of the next statements holds:

1. v has degree 1;
2. $\{v\} = Q_1 \cap Q_2$ for some $Q_1, Q_2 \in \mathcal{C}(G)$;
3. if $u, w \in N(v), uw \in E(G)$, then uw is a multicliqual edge.

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Thank you!