Convex p-partitions and convex p-covers

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November, 2016

Convexity

Given a graph G and a family $\mathcal{C}$ of subsets of $V(G)$, the pair $(G, \mathcal{C})$ is a convexity on G if:

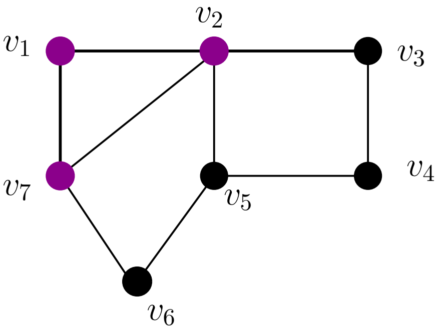
- $\emptyset \in \mathcal{C}$,
- $V(G) \in \mathcal{C}$,
- $\mathcal{C}$ is closed under intersections.

A set of C is called C-convex set.

Geodetic Convexity

$S \subset V(G)$ is a g-convex set if for every minimum length path P of G whose end-vertices belong to S, then every vertex of P belongs to S.

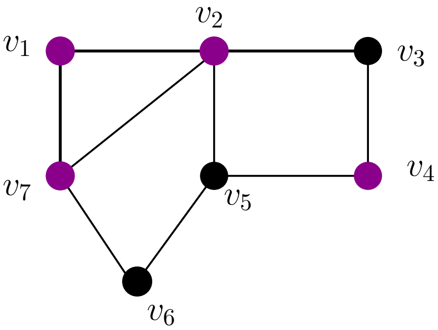
g-convex



Geodetic Convexity

$S \subset V(G)$ is a *g*-convex set if for every minimum length path P of G whose end-vertices belong to S, then every vertex of P belongs to S.

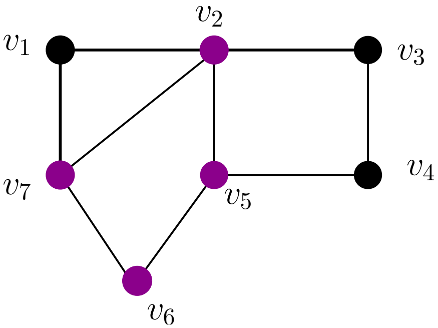
no *g*-convex



Monophonic Convexity

$S \subset V(G)$ is a *m*-convex set if for every induced path P of G whose end-vertices belong to S, then every vertex of P belongs to S.

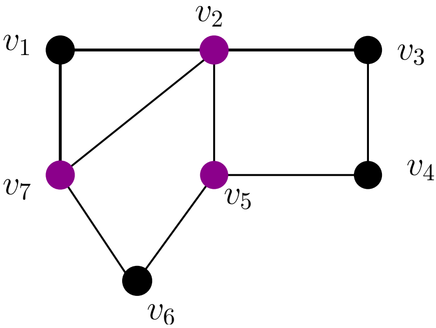
m-convex



Monophonic Convexity

$S \subset V(G)$ is a *m*-convex set if for every induced path P of G whose end-vertices belong to S, then every vertex of P belongs to S.

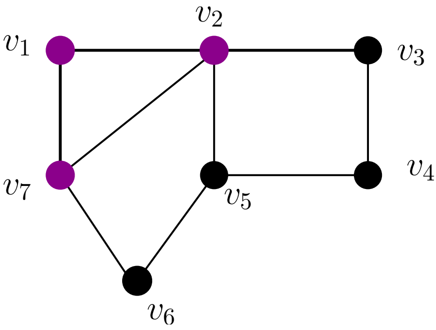
no *m*-convex



P_3-Convexity

$S \subset V(G)$ is a P_3-convex set if for every path of length 2 P of G whose end-vertices belong to S, then every vertex of P belongs to S.

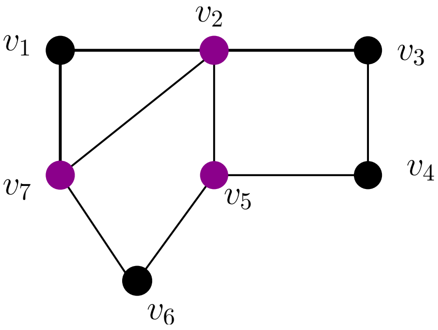
P_3-convex



P_3-Convexity

$S \subset V(G)$ is a P_3-convex set if for every path of length 2 P of G whose end-vertices belong to S, then every vertex of P belongs to S.

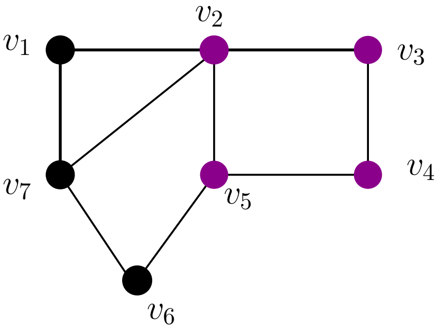
no P_3-convex



P_3^*-Convexity

$S \subset V(G)$ is a P_3^*-convex set if for every induced path of length 2 P of G whose end-vertices belong to S, then every vertex of P belongs to S.

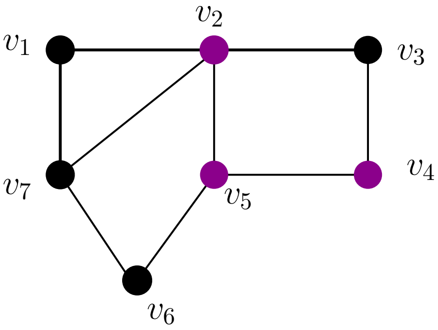
P_3^*-convex



P_3^*-Convexity

$S \subset V(G)$ is a P_3^*-convex set if for every induced path of length 2 P of G whose end-vertices belong to S, then every vertex of P belongs to S.

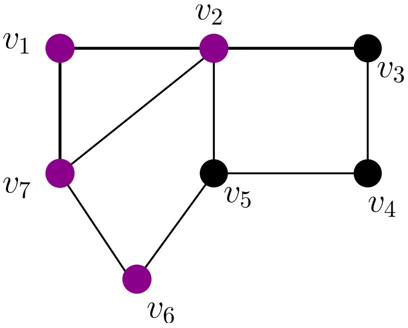
no P_3^*-convex



Digital Convexity

$S \subset V(G)$ is a *d*-convex set if for all $v \in V(G)$: $N_G[v] \subseteq N_G[S]$ then $v \in S$.

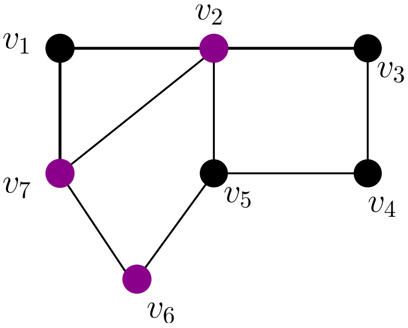
d-convex



Digital Convexity

$S \subset V(G)$ is a *d*-convex set if for all $v \in V(G)$: $N_G[v] \subseteq N_G[S]$ then $v \in S$.

no *d*-convex



Convex cover and convex partition problems

* Given a graph G and a convexity $\mathcal{C}$, G has a convex p-cover if $V(G)$ can be covered by p non-empty proper $\mathcal{C}$-convex sets.

* Given a graph G and a convexity $\mathcal{C}$, G has a convex p-partition if $V(G)$ can be partitioned into p non-empty proper $\mathcal{C}$-convex sets.

Studied problems

In this talk we are going to talk about the complexity of the problems of deciding, given a graph G if there exists a convex p-cover and a convex p-partition of $V(G)$ in the following convexities:

- Monophonic,
- P_3,
- P_3^*,
- Digital.

Problems already studied:

	p-cover, p fixed	p-cover, p variable	p-partition, p fixed	p-partition, p variable
Geodetic	NP-Complete (Buzatu, Cataranciuc)	NP-Complete	NP-Complete (Artigas, Dantas, Dourado, Szwarcfiter, 2011)	NP-Complete
Monophonic	???	???	???	???
Digital	???	???	???	???
P_3	???	???	???	NP-Complete (Centeno, Dantas, Dourado, Rautenbach, Szwarcfiter, 2010)
P_3^*	???	???	???	???

Theorem

Let G be a graph. Deciding if G can be covered by (partition into) 2 m-convex sets can be done in polynomial time.

Monophonic Convexity: cover and partition

Theorem

Let G be a graph. Deciding if G can be covered by (partition into) p m-convex sets is NP-complete for all $p \geqslant 3$.

Theorem

Let G be a graph. Deciding if there is a d-convex cover of $V(G)$ of cardinality at least p is NP-Hard.

Digital Convexity: partition

Theorem

Let G be a bipartite graph. There is a partition into 2 d-convex sets if and only if G has diameter at least 3. Moreover, one such partition can be found in linear time.

Digital Convexity: partition

Theorem

Let G be a graph and $p \geq 2$. Deciding if there exist a d-convex p-partition is NP-hard, even for bipartite planar graphs.

P_3-Convexity: Cover

Theorem

Let G be a graph and $p \geq 2$. Deciding if G can be covered by p P_3-convex sets is NP-complete.

P_3-Convexity: partition

Theorem

Let G be a bipartite graph. Deciding if G can be partition into p P_3-convex sets is NP- complete for all $p \geq 2$.

The same holds for P_3^*-convexity because in bipartite graph the P_3-convex sets and P_3^*-convex sets coincide.

P_3^*-Convexity: cover

Theorem

Let G be a connected bipartite graph and a fixed $p \geq 2$. Deciding if G can be covered by p P_3^*-convex sets is NP-complete.

Summary

	p-cover, p fixed	p-cover, p variable	p-partition, p fixed	p-partition, p variable
Geodetic	NP-Complete	NP-Complete	NP-Complete	NP-Complete
Monophonic	$p = 2$ Polynomial y $p \geqslant 3$ NP-complete	NP-complete	$p = 2$ Polynomial y $p \geqslant 3$ NP-complete	NP-complete
Digital	???	NP-Hard	???	NP-Hard
P_3	???	NP-complete	NP-complete in bipartite graphs	NP-complete
P_3^*	???	NP-complete in connected bipartite graphs	NP-complete in bipartite graphs	NP-complete

Thanks!!!