On Convex Partitions of Graphs in the Triangle Path Convexity

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Summary

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Some Definitions

- Clique - A set of pairwise adjacent vertices.
- Clique Separator - A clique whose removal disconnects the graph.
- Prime Graphs - A graph without a clique separator.

Convexity

Let X be a set. A family C of subsets of X is a convexity if it satisfies the following conditions:

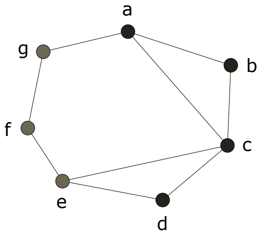
- $\varnothing \in C$ and $X \in C$
- $X_i \cap X_j \in C$, $\forall X_i$ and $X_j \in C$
- For $X_1, X_2, \ldots$ such that $X_1 \subset X_2 \subset \ldots$ with $X_i \in C$ it follows that $X_1 \cup X_2 \cup \ldots \in C$

Convexity

Example: If $V = \{1, 2, 3, 4\}$ and $C = \{\varnothing, \{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3, 4\}\}$, it follows that C is a convexity, because it satisfies es previous conditions.

Paths Convexity

- Many convexities are defined by a set P paths in graphs.
- A path in a graph G is a sequence of distinct vertices $v_1, v_2, v_3, \ldots v_k$ such that $v_i \in V(G)$ for $1 \leq i \leq k$ and $v_i v_{i+1} \in E(G)$ with $1 \leq i < k$.
- ***Example***:



Paths Convexity

Some well studied graph convexities:

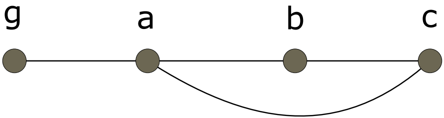
- *Geodetic*,
- *Monophonic*,
- P_3 and
- Triangle-path Convexity

Triangle path Convexity

Let $V(G)$ be the set of vertices of a graph G. The path $P = v_1 \dots v_k$ is a triangle path of a graph G, if there is no edge between vertices v_i and v_j with |j-i|>2.

Triangle path Convexity

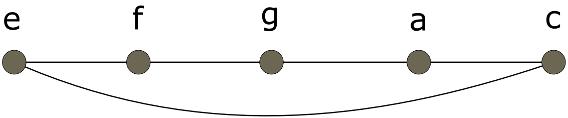
Example: Is the path below a triangle path?



The path $P = gabc$ is a triangle path.

Triangle path Convexity

Example: Is the path below a triangle path?



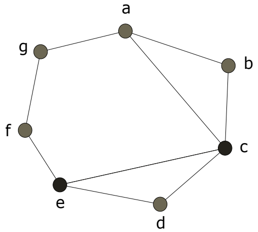
The path $P = efgac$ is not a triangle path.

Set t-Convex

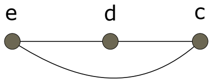
The set $S \subseteq V(G)$ is t-convex if and only if $\forall u, v \in S$ all vertices internal of a triangle path between u and v also belong to S.

Set t-Convex

Given the graph G below, note that the sets $S_1 = \{e, c\}$ and $S_2 = \{e, d, c\}$ are t-convex sets.

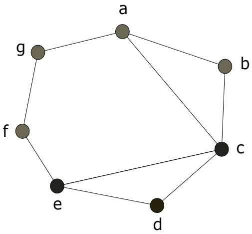


The set $S_1 = \{e, c\}$ is not t-convex, because *edc* is a triangle path and d does not belong to the set $S_1 = \{e, c\}$.

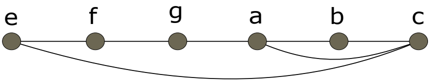


Set t-Convex

Given the graph G below, note that the sets $S_1 = \{e, c\}$ and $S_2 = \{e, d, c\}$ are t-convex sets.



The set $S_2 = \{e, d, c\}$ is t-convex, because every triangle path between vertices of S_2 contains only vertices of S_2.



The Partition into Convex Sets Problem

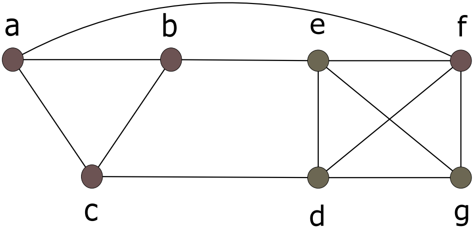
- **Instance**: Graph G.
- **Question**: Can V(G) be partitioned into p disjoint nonempty convex sets

Results of other authors

- Centeno et al. (2010) showed that it is NP-complete to decide if the graph is the p-convex even for graphs *splits* in P_3 Convexity.
- Artigas et al. (2011) showed that it is NP-complete to decide if the graph is p-convex, for to a fixed value $p \geq 2$ in Geodetic Convexity.
- We want to answer what is the complexity of this partition problem for Triangle path Convexity.

The Partition into Convex Sets Problem

- **Theorem**: Let G be a prime graph and S a proper subset of $V(G)$. Then, S is a t-convex set if and only if S is a clique such that every vertex outside S has at most one neighbour in S (DOURADO; SAMPAIO, 2016).



Observations Useful

- **Lemma**: Leb G be a prime graph, and $H = G[N[v]]$. In H:
  - If S is a maximal connected component and not a clique, then each vertex in S must be in a different convex set.
  - If v is not alone in a convex set, then it must be in one of the connected components of H that is clique.
  - So for every isolated clique K in H, we have either $K \cup v$ is a convex partition, or each element of $K \cup v$ is on a different partition.

Observations Useful

- **Corollary 1**: If a clique is large ($> p$ elements), then it must be a convex set.
- **Corollary 2**: We must have at least one clique of size n/p elements.
- **Corollary 3**: Convex sets can only be of type $K \cup v$, and so there are at most n^2 candidate cliques.

Algorithm

- As we saw earlier, we will have at most n^2 candidate cliques.
- We want p t-convex sets that must be cliques.
- So we will have $\binom{n^2}{p}$ different subsets.
- We can try all possibilities in $O(n^{2P})$, which is a polynomial, since p is fixed.

Conclusion and Future Work

- Partitioning a prime graph G in t-convex sets can be done in polynomial time with p-fixed for Triangle path Convexity.
- Can it be done in FPT time?
- What happens when p is part of the input?
- And if the graph G is not prime, what is the complexity of the fixed case?
- If it is NP-hard, we will try to get results for particular cases.
- Other problems for Triangle path Convexity.

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CENTENO, C. C. et al. Convex partitions of graphs induced by paths of order three. *Discrete Mathematics and Theoretical Computer Science*, v. 12, n. 5, p. 175–184, 2010.

DOURADO, M. C.; SAMPAIO, R. M. Complexity aspects of the triangle path convexity. *Discrete Applied Mathematics*, Springer-Verlag, v. 206, p. 39–47, jun 2016. Disponível em: <http://www.sciencedirect.com/science/article/pii/S0166218X16300026>.

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