1-identifying codes on Caterpillar Graphs

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Introduction

Definition - r-identifying code (Karpovsky, Chakrabarty, Levitin, 1998)

Let $G = (V, E)$ be a connected undirected graph, and let $C \subseteq V$ be a subset of V, and let $r \in \mathbb{Z}$.

- C is a **dominating set** in G: $\forall v \in V$, $N[v] \cap C \neq \emptyset$ and,
- C is a r-**separating code** in G: $\forall u \neq v \in V$, $N_{\leq r}[u] \cap C \neq N_{\leq r}[v] \cap C \neq \emptyset$.

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Notation

- $ic(G)$: minimum cardinality of an identifying code of G.
- Elements of C are called **codewords**.

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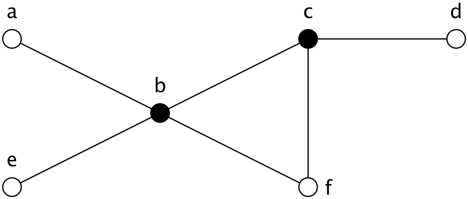

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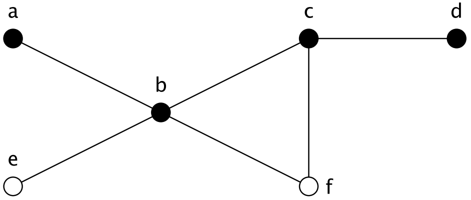
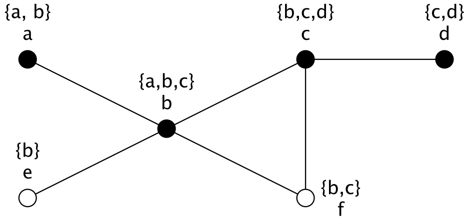


A graph admitting 1-identifying code. Codewords are in black.

Introduction



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Motivation

Applications

- Fault diagnosis in multiprocessors [Karpovsky et al, 1998];
- Location detection in hostile environments [Ray et al, 2003];
- Environmental monitoring [Berger-Wolf et al, 2005];
- Analysis of secondary RNA structures [Haynes et al, 2006].

Motivation

Known Results

- Introduced by [Karpovsky, Chakrabarty, and Levitin, 1998];
- NP-hard problem [Charon, Hudry, and Lobstein, 2003];
- Even for simple classes such as paths, cycles and trees the problem has not been fully solved for r-identifying codes or has only recently been solved.
- Results on 1-identifying codes on paths and cycles only came out in 2004 [Bertrand, Charon, Hudry and Lobstein];
- Cartesian Product of two cliques of the same size [Gravier, Moncel and Senri, 2008];
- Common strategy: to investigate r-identifying codes for restricted values of r.

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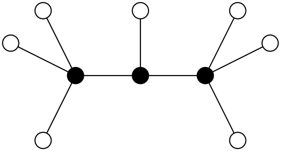
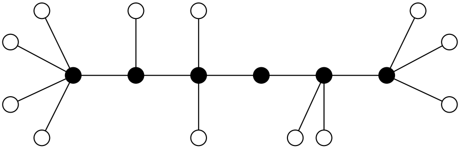
Results

Caterpillars

- A *caterpillar* graph G, or caterpillar tree, is a tree having a chordless path P_s on s vertices, called **central path**, that contains at least one endpoint of every edge.
- Vertices connecting the leaves with the central path are called **support vertices**.

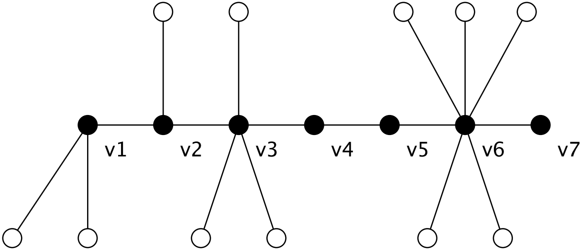




Results

Caterpillar Cat

- A caterpillar tree can be defined as a caterpillar cat $(k_1, k_2, \ldots, k_s)$, which is obtained from a central path $v_1, v_2, \ldots, v_s$ by joining k_i new leaf vertices to v_i, for each $i = 1, 2, \ldots, s$.
- Moreover, $K_1, K_2, \ldots, K_s$ are the set of leaf vertices linked to $v_1, v_2, \ldots, v_s$, respectively.

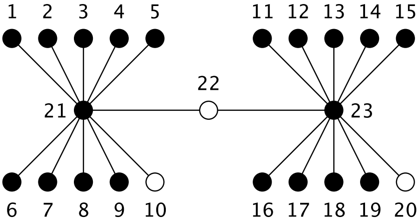


Caterpillar cat (2,1,3,0,0,5,0).

Results

1-identifying codes on trees [Bertrand et al, 2005]

- For general trees on n vertices, any 1-identifying code has at least $3(n+1)/7$ vertices.

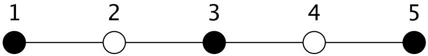


$$ic(T) \geq 11 \text{ but } ic(G) = 20.$$

Results

Theorem 1 (Bertrand et al 2004)

Let $P_s = \{v_1, v_2, \cdots, v_s\}$ be a path on s vertices, $s \geq 3$, and let C be a minimum identifying code in P_s. Then, $|C| = \lceil \frac{s+1}{2} \rceil$.



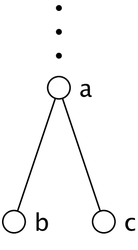
(a) Path P_5

(b) Path P_6

Results

Lemma 2

Let $G = (V, E)$ be a graph, C an identifying code, and $v_i \in V(G)$ be a vertex whose leaves are the set K_i, $|K_i| = k_i, k_i \geq 1$. Then, at least $(k_i - 1)$ leaves of the support vertex v_i are in C.



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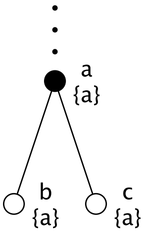

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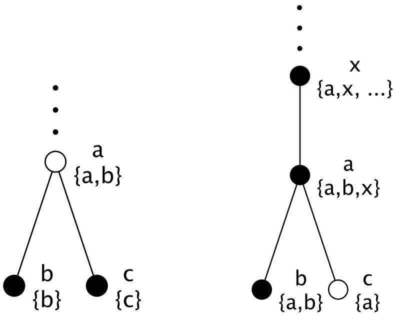

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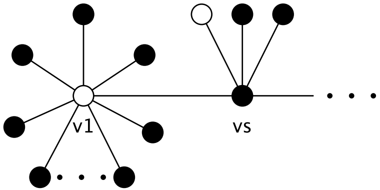
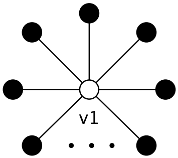


Results

Theorem 3

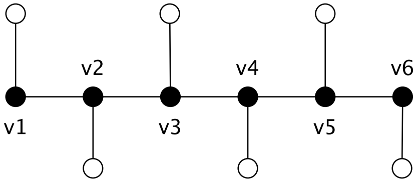
Let G be caterpillar cat (k_1), with $k_1 \geq 2$. Then G is a star and $ic(G) = k_1$.



Results

Theorem 4

Let G be a complete caterpillar $(k_1, k_2, \ldots, k_s)$ on n vertices, $P_s = \{v_1, v_2, \ldots, v_s\}$ be the central path P on $s \geq 3$ vertices, $k_i \geq 1$ for $3 \leq i \leq s$. Then, $ic(G) = l$, where $l = (n - s)$ is the number of leaves in G.



Case $l = s$ (number of leaves is equal to the number of vertices in the central path).

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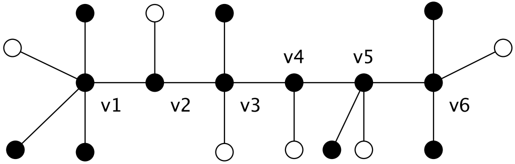
Results



Case $l > s$. Caterpillar cat (4, 1, 2, 1, 2, 3).

Two possibilities:

Case 1: All vertices in K_i are in C, but v_i is not.
Case 2: All vertices in K_i, except one, are in C, and v_i is also in C.

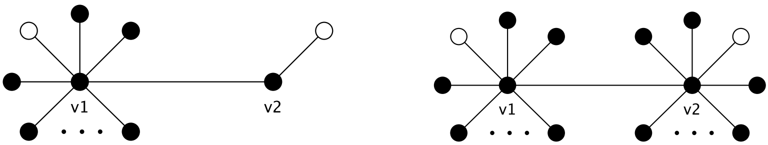
Results

Theorem 5

Let G be a caterpillar cat (k_1, k_s), with $k_1, k_s \geq 1$. Then, $ic(G) = 3$ if $n = 4$ or $ic(G) = k_1 + k_s$.

Proof idea

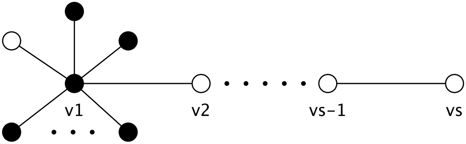
- If $n = 4$, then G is a path on 4 vertices and $ic(G) = \lceil \frac{n+1}{2} \rceil = 3$.
- If $n \geq 5$, the idea is the same applied to the complete caterpillar.



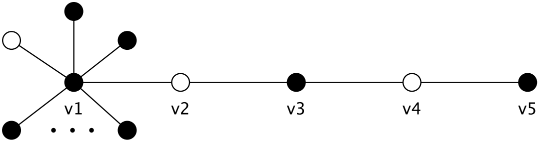
Results

Theorem 6

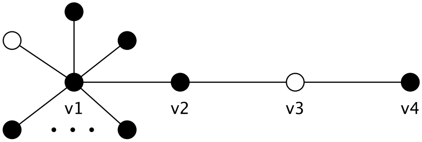
Let G be a caterpillar cat $(k_1, k_2, \ldots, k_s)$, where $k_1 \geq 3$ and $k_i = 0$ for $2 \leq i \leq s$, $s \geq 3$ and let C be a minimum identifying code in G. Then, $ic(G) \geq (k_1 - 1) + ic(P_s)$.



Results



(c) s odd

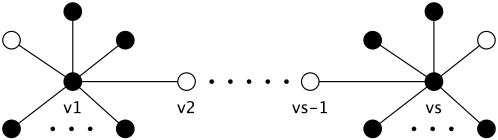


(d) s even

Results

Theorem 7

Let G be a caterpillar cat $(k_1, k_2, \ldots, k_s)$, where $k_1, k_s \geq 3$ and $k_i = 0$ for $2 \leq i \leq (s-1)$, $s \geq 3$ and let C be a minimum identifying code in G. Then, $ic(G) \geq (k_1 - 1) + (k_s - 1) + ic(P_s)$.



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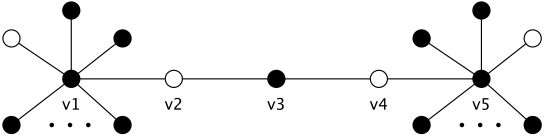
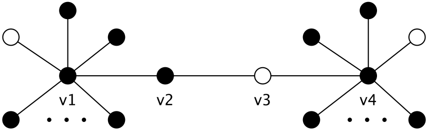


(e) *s* odd



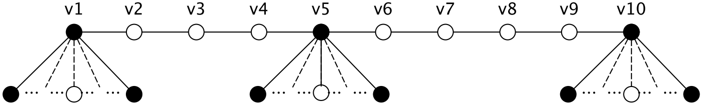
(f) *s* even

Results

Theorem 8

Let G be a caterpillar tree $(v_1, v_2, \ldots, v_s)$ on n vertices, $k_i \geq 3$, P_s be the central path on s vertices, and let $\phi = \{\phi_1, \phi_2, \ldots, \phi_k\}$ be a set of distinct and vertex disjoint subpaths of P_s whose vertices do not have any adjacent vertex that is a leaf. Then,

$$ic(G) \leq (n - s) + \sum_{i=1}^{|\Phi|} \left(\left\lceil \frac{|\phi_i| + 3}{2} \right\rceil - 2 \right).$$



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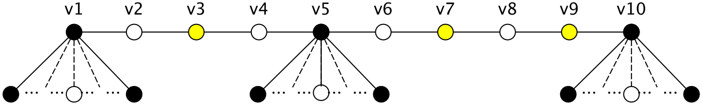

Conclusion and Future Work

We determined the minimum cardinality of a 1-identifying codes on

- complete caterpillars;
- caterpillar cat (k_1) - star, (k_1, k_s), $k_i \geq 1$;
- caterpillar cat $(k_1, \ldots, k_s)$ with $k_1 \geq 3$ and $k_i = 0$ for $2 \leq i \leq s$;
- caterpillar cat $(k_1, \ldots, k_s)$ with $k_1, k_s \geq 3$ and $k_i = 0$ for $2 \leq i \leq (s-1)$;
- and we found an upper bound for general caterpillars.

Future work

- Finish determining 1-identifying codes on caterpillars;
- Investigate r-identifying codes on other classes that have not been characterized yet, such as:
  - Complementary prisms of trees;
  - Complementary prisms of caterpillars;
  - Cartesian product of cycles;
  - Cartesian product of paths.

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