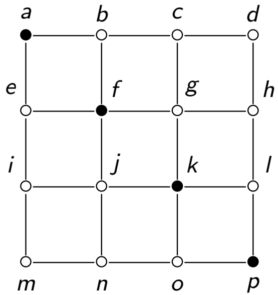
In-Neighbor Convexity in Digraphs

Alessandra A. Pereira Carmen C. Centeno

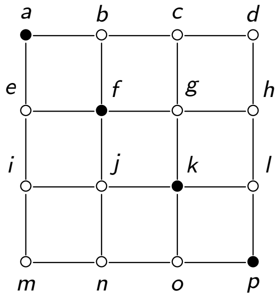
The Motivation

- B. Bollobás
- The spread of infection on a square grid.
- The Art of Mathematics: Coffee Time in Memphis
- P_3 Convexity



The Spread of Infection

1. In a square grid some cells are *infected*.
2. Interactively, a uninfected cell becomes infected if at least two of its neighbors are so.



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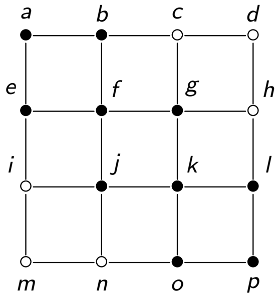

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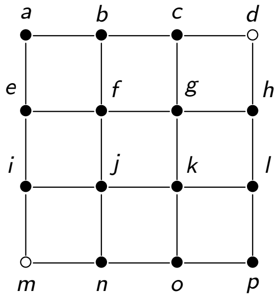

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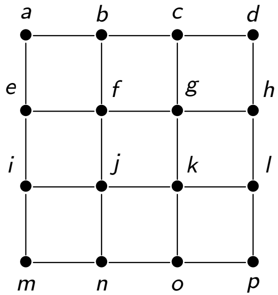

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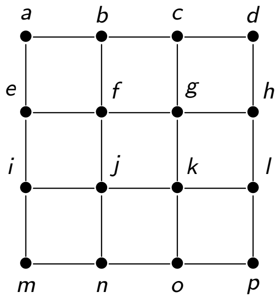

The Spread of Infection

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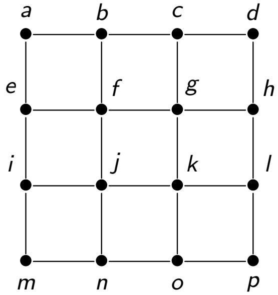
Question

What is the minimum number of originally infected cells to guarantee that all cells of the grid eventually become infected?



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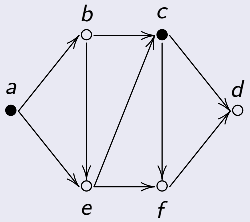
The Spread of infection

P_3 Convexity

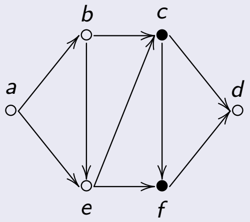
- On the Convexity of Paths of Length Two in Undirected Graphs - Centeno, C.C., Dourado, M.C., Szwarcfiter, J.L.
- Convex Partitions of Graphs induced by Paths of Order Three - Centeno, C.C. , Dantas, S. , Dourado, M.C., Rautenbach, D., Szwarcfiter, J.L.
- Irreversible conversion of graphs - Centeno, C.C., Dourado, M.C., Penso, L.D., Rautenbach, D., Szwarcfiter, J.L.
- Immediate versus Eventual Conversion: Comparing Geodetic and Hull Numbers in P_3 Convexity - Centeno, C.C., Penso, L.D., Rautenbach, D., De Sa, V.G.P.

P_3 Convexity in Digraphs

P_3 Convexity

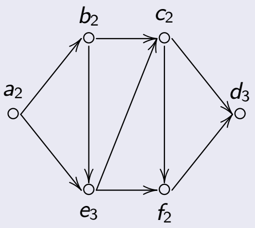


In-Neighbor Convexity



In-Neighbor Convexity

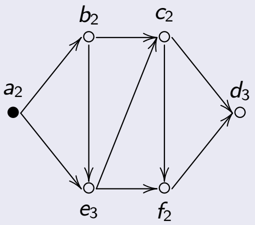
Graph G



$N^{-}(v)$ set of in-neighbors of v.
$f : V(G) \rightarrow \{0, 1, 3, ..., j\}$

In-Neighbor Convexity

Convex Set



$N^{-}(v)$ set of in-neighbors of v
$f : V(G) \rightarrow \{0, 1, 3, ..., j\}$

$\forall v \notin S$ has $|N^{-}(v) \cap S| < f(v)$

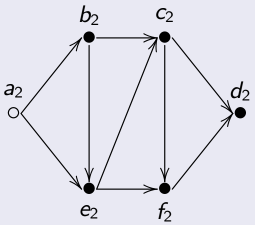
$S = \{a\}$

Parameters of In-Neighbor Convexity

1. Convexity Number $c_p(G)$
2. In-Neighbor Number $p_p(G)$
3. Hull Number $h_p(G)$

Acyclic Digraph

Convexity Number $c_p(G)$



The cardinality of the biggest proper convex set

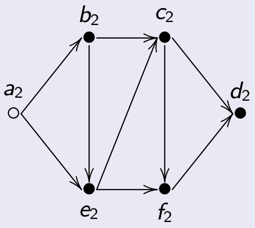
$S = \{b, c, d, e, f\}$
$c_p(G) = 5$

Given G(V,E) an acyclic digraph, $c_p(G)$=

1. $|V(G)| - 1$, se $\exists v : N^{-}(v) < f(v)$,
2. 0, se $\forall v : N^{-}(v) \geq f(v)$.

Acyclic Digraph

Convexity Number $c_p(G)$



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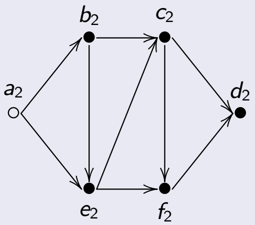

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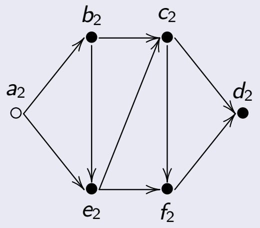

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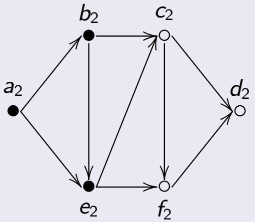
Acyclic Digraph

Interval - $I_p(S)$



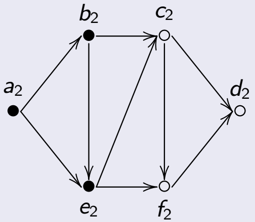
$I_p(S) = S \cup \{\forall v \in V(G) : |N^-(v) \cap S| \geq f(v)\}$

$I_p(\{a, b, e\}) = \{a, b, e, c\}$

When $I_p(S) = V(G)$, S is called in-neighbor set.

Acyclic Digraph

Interval - $I_p(S)$



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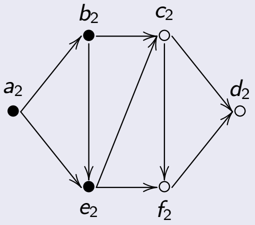

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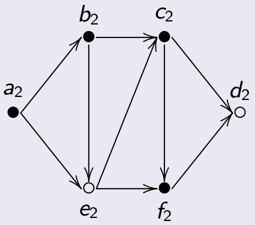
Acyclic Digraph

In-Neighbor Number $p_p(G)$



Cardinality of the smallest in-neighbor set.

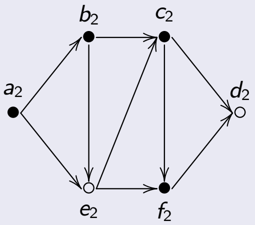
$I_p(\{a, b, c, f\}) = V(G)$

$p_p(G) = 4.$

The In-Neighbor Problem is NP-Complete.

Acyclic Digraph

In-Neighbor Number $p_p(G)$



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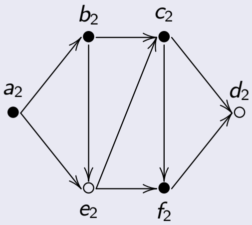

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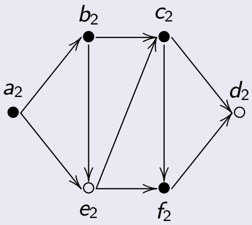

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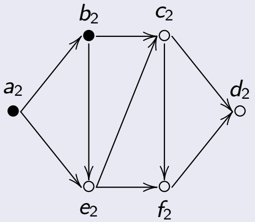

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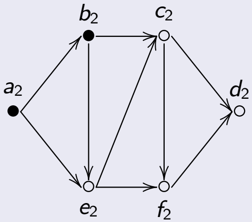
Acyclic Digraph

Hull Number $h_p(G)$



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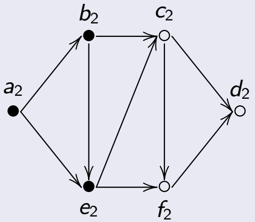
Acyclic Digraph

Hull Number $h_p(G)$



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Acyclic Digraph

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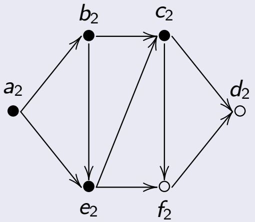

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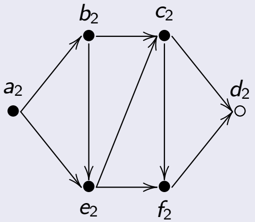

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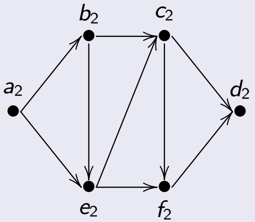

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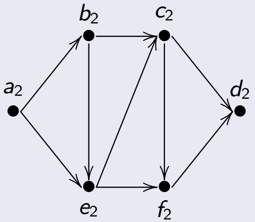
$h_p(G) = 2$

Given an acyclic digraph G, $h_p(G) = |S|$, where S is the set of vertices v with $N^-(v) < f(v)$.

Acyclic Digraph

Hull Number $h_p(G)$



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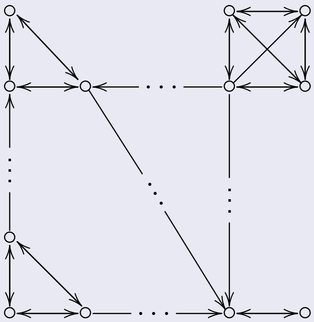
Cyclic Digraph

1. Transitive digraph
2. Cyclic digraph
3. 2-Regular digraph

Cyclic Digraph

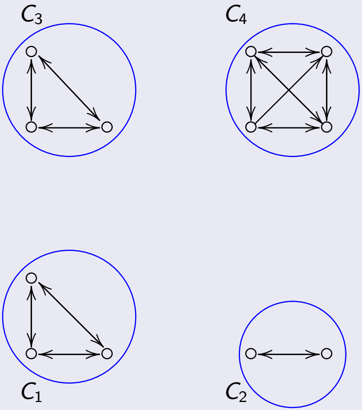
- For cyclic digraph the convexity number is $|V(G)| - 1$, if $\exists v : N^{-}(v) < f(v)$.
- We considered that for all algorithms $\forall v \in V(G) : N^{-}(v) \geq f(v)$.

Convexity Number



Input: Transitive Digraph

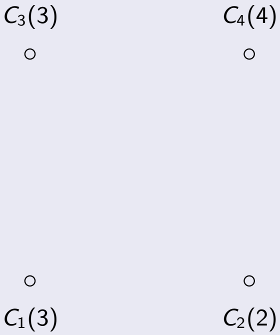
Convexity Number



Input: Transitive Digraph

1. Find strongly connected components.

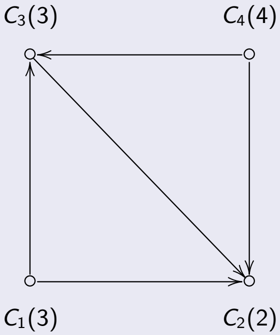
Convexity Number



Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.

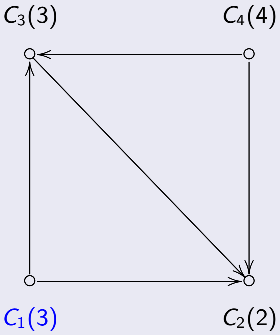
Convexity Number



Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.

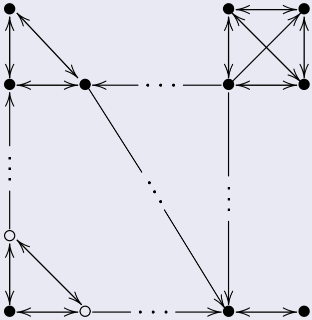
Convexity Number



Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Choose a source of lower weight P_i and do $m := |P_i|$.

Convexity Number

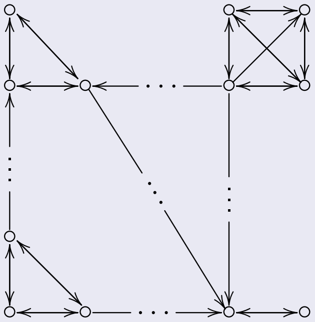


$c_p(G) = 10$

Input: Transitive Digraph

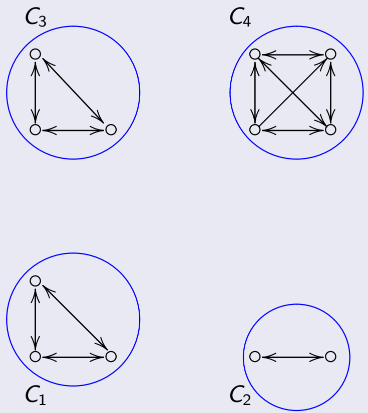
1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Choose a source of lower weight P_i and do $m := |P_i|$.
5. $c_p(G) := |V(G)| - m + f(v) - 1$.

In-Neighbor Number



Input: Transitive Digraph

In-Neighbor Number



Input: Transitive Digraph

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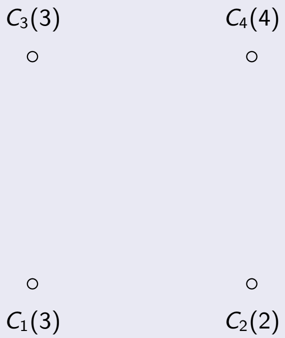
In-Neighbor Number



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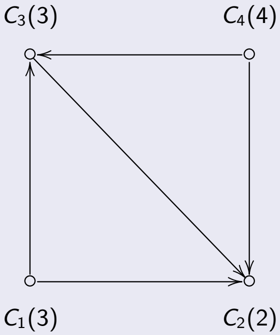

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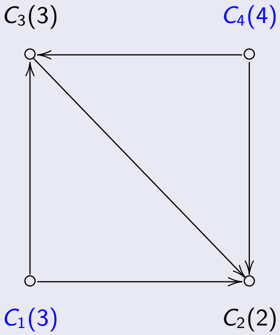
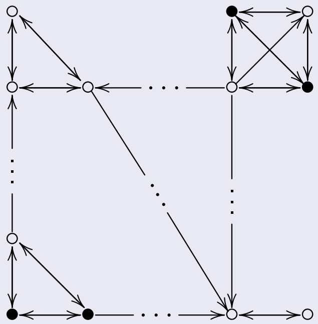


Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).

In-Neighbor Number



$p_p(G) = 4$

Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
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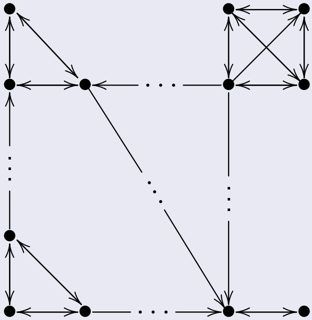
4. Count the number of sources that exist in digraph (q).
5. $p_p(G) := q * f(v)$.

In-Neighbor Number



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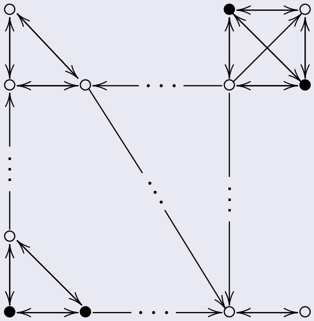
Hull Number



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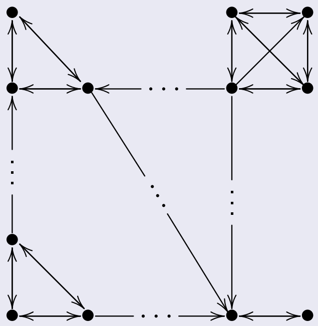
5. $h_p(G) := q * f(v)$.

Hull Number

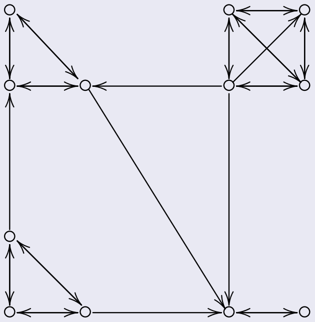


Input: Transitive Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).
5. $h_p(G) := q * f(v)$.

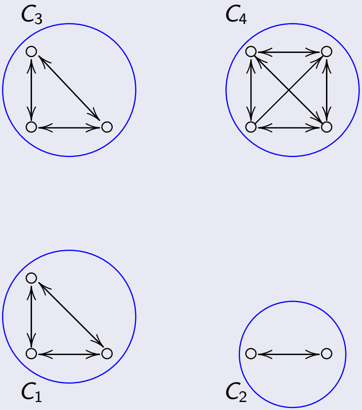
For transitive digraphs $h_p(G) = p_p(G)$.

Convexity Number



Input: Cyclic Digraph

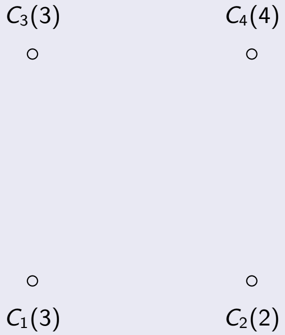
Convexity Number



Input: Cyclic Digraph

1. Find strongly connected components.

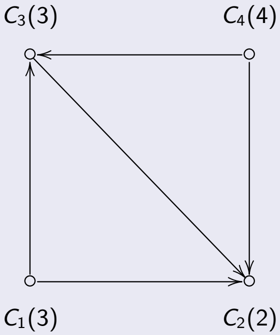
Convexity Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.

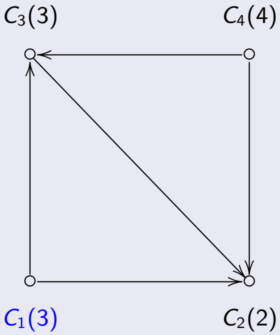
Convexity Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.

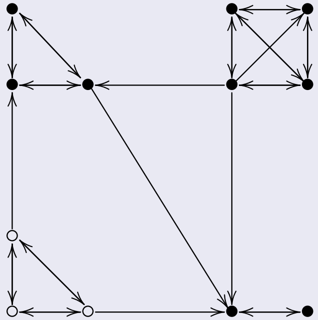
Convexity Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Choose a source of lower weight P_i and do $m := |P_i|$.

Convexity Number

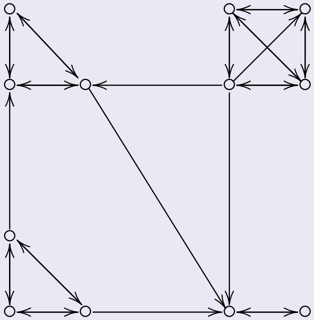


$c_p(G) = 9$

Input: Cyclic Digraph

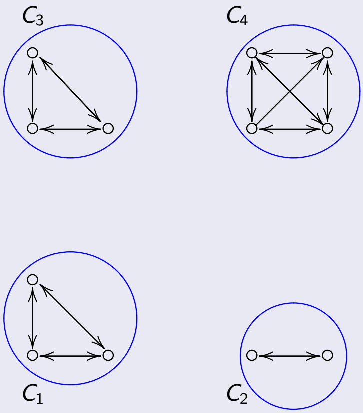
1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Choose a source of lower weight P_i and do $m := |P_i|$.
5. $c_p(G) := |V(G)| - m$.

Hull Number



Input: Cyclic Digraph

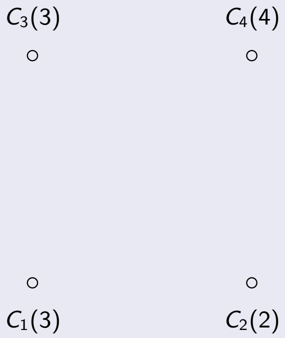
Hull Number



Input: Cyclic Digraph

1. Find strongly connected components.

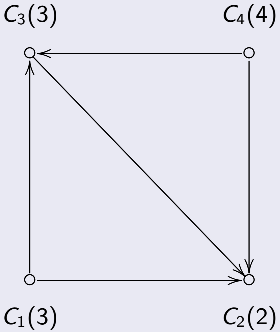
Hull Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.

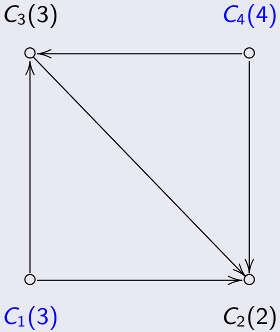
Hull Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.

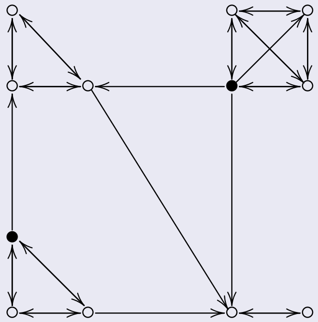
Hull Number



Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).

Hull Number

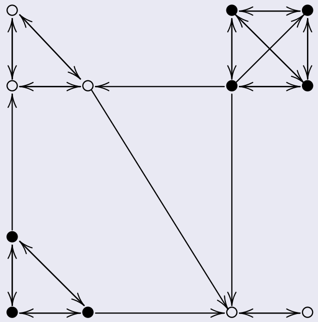


$h_p(G) = 2$

Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).
5. $c_p(G) := q$.

Hull Number

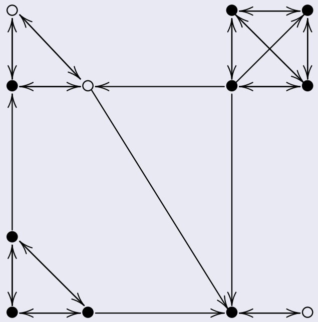


$h_p(G) = 2$

Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).
5. $c_p(G) := q$.

Hull Number

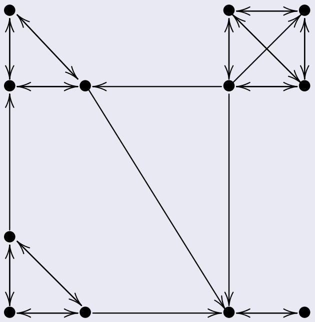


$h_p(G) = 2$

Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).
5. $c_p(G) := q$.

Hull Number



$h_p(G) = 2$

Input: Cyclic Digraph

1. Find strongly connected components.
2. Reduction of strongly connected components.
3. Connect C_is.
4. Count the number of sources that exist in digraph (q).
5. $c_p(G) := q$.

Conclusions

1. Transitive digraph: convexity number, in-neighbor number and hull number.
2. Cyclic digraph: convexity number and hull number.
3. 2-Regular digraph: convexity number.