Álgebras de Hopf punteadas sobre grupos finitos simples de tipo Lie

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Joint work with N. Andruskiewitsch and G. Carnovale.
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We say that a finite group \( G \) \textit{collapses} when every finite-dimensional pointed Hopf algebra \( H \), with \( G(H) \cong G \) is isomorphic to \( \mathbb{k}G \).
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- For $G$ not simple, non-trivial examples also exists, among others: $S_3$ [AHS], $S_4$ [GG], $D_{4t}$ for $t \geq 3$ [FG], $G$ associated to an affine rack [GIV].
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Let $0 = H_{-1} \subset H_0 = \mathbb{k}G(H) \subset H_1 \subset \ldots$ be the coradical filtration of $H$ and $\text{gr} \ H = \bigoplus_{n \in \mathbb{N}_0} H_n/H_{n-1} \cong R\# \mathbb{k}G(H)$. 
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Determine all $V \in \mathbb{k}G \mathcal{YD}$ with $\dim \mathcal{B}(V) < \infty$. 
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Let $0 = H_{-1} \subset H_0 = kG(H) \subset H_1 \subset \ldots$ be the coradical filtration of $H$ and $\text{gr } H = \bigoplus_{n \in \mathbb{N}_0} H_n/H_{n-1} \simeq R \# kG(H)$.

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- For every $V \in kG\mathcal{YD}$, $\dim \mathcal{B}(V) = \infty$.
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The following are equivalent [AFGV1]:

- $G$ collapses.
- For every $V \in k G \mathcal{YD}$, $\dim \mathfrak{B}(V) = \infty$.
- For every irreducible $V \in k G \mathcal{YD}$, $\dim \mathfrak{B}(V) = \infty$. 

Fact:

All irreducible Yetter-Drinfeld modules over $kG$ are of the form $M(O, \rho) = \text{Ind}_G^C_G(g)\rho$, where $O$ is a conjugacy class of $G$ and $\rho \in \text{Irr}(C_G(g))$ for $g \in O$ fixed.

Set $B(O, \rho) := B(M(O, \rho))$.

Question: Determine all pairs $(O, \rho)$ with $\dim B(O, \rho) < \infty$.

Crucial: $B(O, \rho)$ depends only on the underlying braided vector space $(kO, c\rho)$, i.e., $B(O, \rho)$ depends only on the rack $O$ and the non-principal 2-cocycle arising from $\rho$.

Question [AFGV1]: Determine all pairs $(X, q)$, where $X$ is a finite rack and $q$ is a non-principal 2-cocycle, such that $\dim B(X, cq) < \infty$. 
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(a) $x \triangleright$ is a bijection for any $x \in X$,
(b) $x \triangleright (y \triangleright z) = (x \triangleright y) \triangleright (x \triangleright z)$ for all $x, y, z \in X$.

The archetypical example of a rack is a conjugacy class in a group $G$, with $x \triangleright y = xyx^{-1}$ for all $x, y \in G$.

We say that a rack is:

- abelian if $x \triangleright y = y$ for all $x, y \in X$.
- decomposable if it contains two subracks $R, S$ such that $X = R \cupdot S$ and $R \triangleright S \subseteq S, S \triangleright R \subseteq R$.
- simple if $|X| > 1$ and any rack epimorphism $X \twoheadrightarrow Y$ is bijective or $|Y| = 1$. 
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- **Type F** if it has a family of mutually disjoint subracks $(R_a)_{a \in A}$ such that $R_a \triangleright R_b = R_b$ for all $a, b \in A$; for all $a \neq b \in A$, there are $r_a \in R_a$, $r_b \in R_b$ such that $r_a \triangleright r_b \neq r_b$ and $A$ has four elements.
Definition

A rack $X$ collapses when $\dim \mathcal{B}(X, q) = \infty$ for every finite faithful 2-cocycle $q$.

Therefore, to solve the initial question we first need to determine all conjugacy classes in $G$ that collapse. We have criteria that help to solve the problem without looking at the 2-cocycle. We say that a rack $X$ is of

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- **Type C** if it has a decomposable subrack $Y = R \coprod S$, where $|R| > 6$ or $|S| > 6$, with elements $r \in R$, $s \in S$ such that $r \rhd s \neq s$. 
Álgebras de Hopf punteadas sobre grupos finitos simples de tipo Lie

Racks

Properties

Remarks:

▶ Si $O$ es una clase de conjugación en un grupo finito $G$, entonces $O$ es de tipo D si y solo si existen $x, y \in O$ que no se conjugan en $\langle x, y \rangle$ y $(xy)^2 \neq (yx)^2$.

▶ Si $Z$ es un rack finito que admite una morfismo de rack $Z \twoheadrightarrow X$, donde $X$ es de tipo D (F, C), entonces $Z$ es de tipo D (F, C).

▶ Si $Z$ es indecomponible, entonces admite una morfismo de rack $Z \twoheadrightarrow X$ con $X$ simple.

Teorema [AFGV1], [H], [ACG]

Un rack $X$ de tipo C, D o F se colapsa.

▶ Un rack $X$ es cthulhu cuando no es de tipo C, D, F.

▶ Un rack $X$ es sober cuando cada subrack es abeliano o indecomponible. Un rack sober es cthulhu.
Remarks:

- If $O$ is a conjugacy class in a finite group $G$, then $O$ is of type D if and only if there exist $x, y \in O$ such that $x, y$ are not conjugated in $\langle x, y \rangle$ and $(xy)^2 \neq (yx)^2$.

- If $Z$ is a finite rack that admits a rack epimorphism $Z \twoheadrightarrow X$, where $X$ is of type D (F, C), then $Z$ is of type D (F, C).

- If $Z$ is indecomposable, then it admits a rack epimorphism $Z \twoheadrightarrow X$ with $X$ simple.

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A rack $X$ of type C, D or F collapses.

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There are three families of finite simple groups of Lie type, according to the classes of Steinberg endomorphisms:
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*Chevalley groups.* Correspond to $\mathbb{F}_q$-split Steinberg maps:

\[
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\[
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\[
PSp_{2n}(q), \quad n \geq 2; \]
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If $x = x_s x_u$ is the Chevalley-Jordan decomposition in $G$, then $x_s, x_u \in G$. Let $K = C_G(x_s)$, a reductive subgroup of $G$, and $K = K \cap G = C_G(x_s)$. Since $x_u \in K$, $O_{K x_u}$ is a subrack of $O_{G x}$ and we can reduce our study to the case when $x$ is either unipotent or semisimple.
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Since $x_u \in K$, $O^K_{x_u}$ is a subrack of $O^G_x$ and we can reduce our study to the case when $x$ is either unipotent or semisimple.
For $a \in (\mathbb{F}_q^n)^{n-1}$, define

$$r_a = \begin{pmatrix}
1 & a_1 & 0 & \ldots & 0 \\
0 & 1 & a_2 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \ldots & \ldots & 1 & a_{n-1} \\
0 & \ldots & \ldots & 0 & 1
\end{pmatrix}.$$
For $a \in (\mathbb{F}_q^{\ast})^{n-1}$, define

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0 & \ldots & 0 & 1 & a_{n-1} \\
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\end{pmatrix}.$$ 

A unipotent element $u \in \text{GL}_n(q)$ is of type $\lambda = (\lambda_1, \ldots, \lambda_k)$ if it is conjugate to the element

$$u = \begin{pmatrix}
u_1 & 0 & \ldots & 0 \\
0 & u_2 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & \ldots & \ldots & u_k
\end{pmatrix} \quad \text{where} \quad u_i = r_1 \in \mathbb{F}_q^{\lambda_i \times \lambda_i}.$$
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**Theorem**

Let $O$ be a unipotent conjugacy class in $G$. If $O$ is not listed below, then it collapses.

<table>
<thead>
<tr>
<th>$n$</th>
<th>type</th>
<th>$q$</th>
<th>Remark</th>
</tr>
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<tbody>
<tr>
<td>2</td>
<td>(2)</td>
<td>even or not a square</td>
<td>sober</td>
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<tr>
<td>3</td>
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<td></td>
<td>(2, 1)</td>
<td>2</td>
<td>cthulhu</td>
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**Proposition**

Let $x \in G = \text{SL}_n(q)$ with Chevalley-Jordan decomposition $x = x_s x_u$. Assume that $x_s$ is not central and $x_u \neq e$. Then $O^G_x$ collapses.
For non-semisimple and non-unipotent classes in \( SL_n(q) \) we have the following

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Nevertheless, for \( G = PSL_n(q) \) we do not have the complete result yet:
For non-semisimple and non-unipotent classes in $\text{SL}_n(q)$ we have the following

**Proposition**

Let $x \in G = \text{SL}_n(q)$ with Chevalley-Jordan decomposition $x = x_s x_u$. Assume that $x_s$ is not central and $x_u \neq e$. Then $O^G_x$ collapses.

Nevertheless, for $G = \text{PSL}_n(q)$ we do not have the complete result yet:

**Proposition**

Let $x \in \text{SL}_n(q)$ with Chevalley-Jordan decomposition $x = x_s x_u$. Assume that $x_s$ is not central and $x_u \neq e$. If $x_u$ is not listed below, then $O^G_{x_u}$ collapses. In consequence, if $x = \pi(x) \in G$, then $O^G_x$ collapses.
<table>
<thead>
<tr>
<th>[n = h_1 \Lambda_1 + \cdots + h_\ell \Lambda_\ell]</th>
<th>[x_u = (u_1, \ldots, u_\ell)]</th>
<th>[q = (q^{\mu_1}, \ldots, q^{\mu_\ell})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[n = 2\Lambda_1 &gt; 2, \ \ell = 1]</td>
<td>[x_u = u_1]</td>
<td>all</td>
</tr>
<tr>
<td>[h_1 = 2]</td>
<td>[(u_1, \text{id}, \ldots, \text{id})]</td>
<td>odd and 9 or not a square</td>
</tr>
<tr>
<td>[h_i \geq 2 \text{ for } 2 \leq i \leq \ell]</td>
<td>[u_i = \text{id for } i \neq 1]</td>
<td></td>
</tr>
<tr>
<td>[h_j = 2]</td>
<td>[(u_1, \ldots, u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q = 3]</td>
</tr>
<tr>
<td>[#{j : u_j \neq \text{id}} \geq 2]</td>
<td>[u_i = \text{id for } j &lt; i \leq \ell]</td>
<td></td>
</tr>
<tr>
<td>[h_i \geq 2 \text{ for } j &lt; i \leq \ell]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[h_1 = 2]</td>
<td>[(u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q = 3]</td>
</tr>
<tr>
<td>[h_1 = 3]</td>
<td>[(u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q = 2]</td>
</tr>
<tr>
<td>[h_1 = 4]</td>
<td>[(u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q = 2]</td>
</tr>
<tr>
<td>[u_1 \text{ of type } (2, 1, 1)]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[h_j = 2]</td>
<td>[(u_1, \ldots, u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q = 2]</td>
</tr>
<tr>
<td>[#{j : u_j \neq \text{id}} \geq 2]</td>
<td>[u_i = \text{id for } j &lt; i \leq \ell]</td>
<td></td>
</tr>
<tr>
<td>[h_1 = 2,]</td>
<td>[(u_1, \text{id}, \ldots, \text{id})]</td>
<td>[q \text{ even}]</td>
</tr>
</tbody>
</table>
Now we summarize the results (still on progress) on collapsing unipotent classes in a Chevalley group.
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Now we summarize the results (still on progress) on collapsing unipotent classes in a Chevalley group. Let $G$ be a Chevalley group, $G \neq \text{PSL}_n(q)$.

**Theorem**

*Let $O$ be a unipotent conjugacy class in $G$. If $O$ is not listed below, then it collapses.*
<table>
<thead>
<tr>
<th>$G$</th>
<th>$q$</th>
<th>type or representative</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{PSp}_{2n}(q)$, $n \geq 2$</td>
<td>even, odd &amp; $\not\equiv \Box$</td>
<td>all, $(1^r_1, 2)$</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>$(1^r_1, 2)$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$(1^r_1, 2^r_2, 3^r_3)$, $r_2 r_3 &gt; 0$</td>
</tr>
<tr>
<td>$\text{P} \Omega_{2n+1}(q)$, $n \geq 3$</td>
<td>3</td>
<td>$(1^r_1, 2^r_2, 3^r_3)$, $r_2 r_3 &gt; 0$</td>
</tr>
<tr>
<td>$\text{P} \Omega_{2n}^+(q)$, $n \geq 4$</td>
<td>even</td>
<td>all, $(1^r_1, 2^r_2, 3^r_3)$, $r_2 r_3 &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>$(1^r_1, 2^r_2, 3^r_3)$, $r_2 r_3 &gt; 0$</td>
</tr>
<tr>
<td>$E_6(q)$</td>
<td>2, 4</td>
<td>all except $x_{\alpha_1}(1)$</td>
</tr>
<tr>
<td>$E_7(q)$</td>
<td>2</td>
<td>all except $y_{119}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>all except $y_{113}$, $y_{115}$, $y_{117}$, $y_{118}$, $y_{119}$</td>
</tr>
<tr>
<td>$E_8(q)$</td>
<td>2</td>
<td>all except $z_{195}$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>all except $z_{189}$, $z_{193}$, $z_{194}$, $z_{195}$</td>
</tr>
<tr>
<td></td>
<td>$p=2, 3, 5$</td>
<td>$\subset D_8(a_7)$</td>
</tr>
<tr>
<td>$F_4(q)$</td>
<td>2, 3, 4</td>
<td>all except $x_4$</td>
</tr>
</tbody>
</table>
GRACIAS


S. Freyre, M. Graña and L. Vendramin, *On Nichols algebras over $\text{PGL}(2, q)$ and $\text{PSL}(2, q)$*. J. Algebra Appl. 9 (2010), 195–208.


