Perturbation and spectral theory for J-non-negative operators and applications

Carsten Trunk, TU Ilmenau, Germany

We present recent developments for J-non-negative operators in spaces with an indefinite metric. More precisely, we consider a Hilbert space $\mathcal{H}$ with positive definite inner product $(\cdot,\cdot)$ and a self-adjoint, bounded operator J with $J^2 = I$ which serves as the Gramian of

$$[x, y] := (Jx, y) \quad \text{for } x, y \in \mathcal{H}.$$

It is usual, to call the tuple $(\mathcal{H}, [\cdot,\cdot])$ a *Krein space*. A densely defined operator in $\mathcal{H}$ is called *J-non-negative*, if it has a non-empty resolvent set and satisfies for all x in its domain

$$[Ax, x] \geq 0.$$

It is well-known that the spectrum of such an operator is real and that there exists a spectral function with (possible) singularities at 0 and ∞, see [1, 2, 5]. In the talk we will discuss various spectral properties like Jordan chains or the numerical range. Main focus of the talk is the description of the spectrum after a one-dimensional perturbation.

J-non-negative operators appear in the study of operator polynomials (e.g. [3, 4]) and indefinite Sturm-Liouville equations, see, e.g. [6]. Currently much effort is devoted to inverse spectral problems related to left-definite Sturm Liouville problems and the Camassa-Holm equation.

References

[1] T. Ando, *Linear Operators in Krein Spaces*, Sapporo, Japan 1979.

[2] T.Ya. Azizov and I.S. Iokhvidov, *Linear Operators in Spaces with an Indefinite Metric*, John Wiley and Sons, Chichester, New York, 1989.

[3] J. Bognar, *Indefinite Inner Product Spaces*, Springer, 1974.

[4] M.G. Krein, *Introduction to the theory of indefinite J-spaces and to the theory of operators in those spaces*, Amer. Math. Soc. Transl. (2) 93 (1970), 103–176

[5] H. Langer, *Spectral functions of definitizable operators in Krein spaces*, Lecture Notes in Mathematics 948, Springer (1982), 1–46.

[6] A. Zettl, *Sturm-Liouville Theory*, AMS, Providence, RI, 2005.