Dynamical Sampling and Bessel Orbits of Operators

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Dynamical Sampling aims to subsample solutions of linear dynamical systems at various times. In a real world scenario one might think of sensors on the ground measuring temperature or other quantities. Instead of installing a dense network of sensors, the idea is to use less sensors and to exploit the dynamical process which the quantity is subject to. One way to model this consists of considering inner products (the samples) of the form $\langle h, A^n f_i \rangle$, where h is the signal (temperature distribution etc.), $(f_i)_{i\in I}$ a system of fixed vectors, and A a linear operator which is connected with the dynamical system. Here, we restrict ourselves to normal operators A since they provide a rich spectral theory. The objective of the general Dynamical Sampling problem is to find out for which operators A and which families $(f_i)_{i\in I}$ the (arbitrary) signal h can be stably recovered from the sampling data, i.e., the system $(A^n f_i)_{n\in\mathbb{N},\, i\in I}$ is a frame for the underlying Hilbert space.

For this, it is of course necessary that for each $i \in I$ the system $(A^n f_i)_{n\in\mathbb{N}}$ is a Bessel sequence. We first address this problem and give various characterizations. It turns out that the question is highly dependent on the interplay of the spectral measure of the operator A and the vector f_i. In our treatment of the general problem we only consider finite index sets I. In this case, the operator A is necessarily a diagonal operator with eigenvalues λ_j of multiplicity at most $|I|$ in the open unit disk. Moreover, the sequence $(\lambda_j)_{j\in\mathbb{N}}$ must be a finite union of so-called *uniformly separated* sequences. We will complete this list of conditions to a characterization of the problem and deduce some consequences. We also give an upper bound on the number of uniformly separated subsequences in the sequence $(\lambda_j)_{j\in\mathbb{N}}$.

The talk is based on joint work with C. Cabrelli, U. Molter, and V. Paternostro (all from UBA).